

1) Determine if the function has a vertical asymptote or a removable discontinuity at $x = -1$.

$$1b) f(x) = \frac{x^2 + 1}{x + 1}$$

$$\lim_{x \rightarrow -1^+} \frac{x^2 + 1}{x + 1} = \infty$$

$$\lim_{x \rightarrow -1^-} \frac{x^2 + 1}{x + 1} = -\infty$$

Vertical asymptote at $x = -1$

$$1c) f(x) = \frac{\sin(x+1)}{x+1}$$

$$f(x) = \frac{\sin(x+1)}{x+1}$$

$$\lim_{x \rightarrow -1} \frac{\sin(x+1)}{x+1} = 1$$

Removable Discontinuity

$$2c) \lim_{x \rightarrow 2^+} \frac{x}{x-2}$$

$$= \lim_{x \rightarrow 2^+} x \cdot \lim_{x \rightarrow 2^+} \frac{1}{x-2}$$

$$= 2 \cdot (+\infty)$$

$$= +\infty$$

$$2d) \lim_{x \rightarrow 1^+} \frac{2+x}{1-x}$$

$$= \lim_{x \rightarrow 1^+} (2+x) \cdot \lim_{x \rightarrow 1^+} \frac{1}{1-x}$$

$$= 3 \cdot \lim_{x \rightarrow 1^+} \frac{-1}{x-1}$$

$$= 3 \cdot (-\infty)$$

$$= -\infty$$

$$2e) \lim_{x \rightarrow -3^-} \frac{x+3}{(x^2+x-6)} = \lim_{x \rightarrow -3^-} \frac{x+3}{(x+3)(x-2)}$$

$$= \lim_{x \rightarrow -3^-} \frac{1}{x-2} = -\frac{1}{5}$$

$$2f) \lim_{x \rightarrow 0^-} \left(1 + \frac{1}{x}\right)$$

$$= \lim_{x \rightarrow 0^-} 1 + \lim_{x \rightarrow 0^-} \frac{1}{x}$$

$$= 1 + (-\infty)$$

$$= -\infty$$

$$2g) \lim_{x \rightarrow 0^+} \frac{2}{\sin x}$$

$$= \lim_{x \rightarrow 0^+} 2 \csc x$$

$$= +\infty$$

