1b) $f'(t) = -4t + 3$

1c) $y = \dfrac{1}{x^5} = x^{-5}$

$y' = -5x^{-6} = -\dfrac{5}{x^6}$

1d) $y = \sqrt[5]{x} = x^{1/5}$

$y' = \dfrac{1}{5}x^{-4/5} = \dfrac{1}{5x^{4/5}}$

2b) $y' = 2x + \frac{1}{2}\sin x$

2c) $y' = -\dfrac{1}{x^2} - 3\cos x$

2d) $y = \dfrac{6}{(5x)^3}$

$y = \dfrac{6}{125}x^{-3}$

$y' = -\dfrac{18}{125}x^{-4}$

$y' = -\dfrac{18}{125x^4}$

3b) $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2} = x - 3 + 4x^{-2}$

$f'(x) = 1 - \dfrac{8}{x^3} = \dfrac{x^3 - 8}{x^3}$

3c) $y = x(x^2 + 1) = x^3 + x$

$y' = 3x^2 + 1$

3d) $f(x) = \sqrt{x} - 6\sqrt[3]{x} = x^{1/2} - 6x^{1/3}$

$f'(x) = \dfrac{1}{2}x^{-1/2} - 2x^{-2/3} = \dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^{2/3}}$

4b) $f(x) = \dfrac{2}{\sqrt[4]{x^3}} = 2x^{-3/4}$

$f'(x) = -\dfrac{3}{2}x^{-7/4} = -\dfrac{3}{2x^{7/4}}$

At $(1, 2)$: $f'(1) = -\dfrac{3}{2}$

Tangent line: $y - 2 = -\dfrac{3}{2}(x - 1)$

5b) $y = x^3 + x^2$

$y' = 3x^2 + 2x$

$0 = 3x^2 + 2x$

$0 = x(3x + 2)$

$x = 0$ or $x = -2/3$

Horizontal tangents (0, 0), (-2/3, 4/27)

5c) $y = \frac{1}{x^2} = x^{-2}$

$y' = -2x^{-3} = -\frac{2}{x^3}$ cannot equal zero.

Therefore, there are no horizontal tangents.

5d) $y = x + \sin x, 0 \leq x < 2\pi$

$y' = 1 + \cos x = 0$

$\cos x = -1 \Rightarrow x = \pi$

At $x = \pi$: $y = \pi$

Horizontal tangent: (π, π)

6b)

$\frac{k}{x} = -\frac{3}{4}x + 3$ Equate functions.

$-\frac{k}{x^2} = -\frac{3}{4}$ Equate derivatives.

So, $k = \frac{3}{4}x^2$ and

$$\frac{\frac{3}{4}x^2}{x} = -\frac{3}{4}x + 3 \Rightarrow \frac{3}{4}x = -\frac{3}{4}x + 3$$

$$\Rightarrow \frac{3}{2}x = 3 \Rightarrow x = 2 \Rightarrow k = 3.$$

7) AP MULTIPLE CHOICE EXAMPLES

1) If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$

(A) -6 (B) -3 (C) 3 (D) 6 (E) 8

$f'(x) = \frac{3}{2} x^{\frac{1}{2}}$

$f'(4) = \frac{3}{2}(4)^{\frac{1}{2}}$

$f'(4) = \frac{3}{2} \cdot \sqrt{4} = \frac{3}{\cancel{2}} \cdot \cancel{2} = 3$

2) If $f(x) = x + \sin x$, then $f'(x) =$

(A) $1 + \cos x$ (B) $1 - \cos x$ (C) $\cos x$

(D) $\sin x - x\cos x$ (E) $\sin x + x\cos x$

Derivative of x is 1

Derivative of is $\cos x$

We can seperate and differentiate sums & differences then, recombine as one answer

3) If $f(x) = \sin x$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

$f'(x) = \cos x$

$f'\left(\frac{\pi}{3}\right) = \cos \frac{\pi}{3}$

$f'\left(\frac{\pi}{3}\right) = \frac{1}{2}$

4) If the line $3x-4y=0$ is tangent in the first quadrant to the curve $y=x^3+k$, then k is

$y' = 3x^2 + 0$

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) $-\frac{1}{8}$ (E) $-\frac{1}{2}$

$3x-4y=0$ is the tangent line to $y=x^3+k$

or

$y=\frac{3}{4}x$

has a slope of $3/4$

If $y=\frac{3}{4}x$ meets $y=x^3+k$

then $\frac{3}{4}x = x^3+k$

$\frac{3}{4}\cdot\frac{1}{2} = \left(\frac{1}{2}\right)^3+k$

$\frac{3}{8} = \frac{1}{8}+k$

$\frac{2}{8}=k$

$\frac{1}{4}=k$

If $y=\frac{3}{4}x$ has slope "$\frac{3}{4}$",

then $3x^2 = \frac{3}{4}$

y'

$x^2=\frac{1}{4}$ $x=\pm\sqrt{\frac{1}{4}}$

$x=+\frac{1}{2}$

1st QUAD

5) If $f(x)=e^x$, which of the following is equal to $f'(e)$?

(A) $\lim\limits_{h\to 0}\frac{e^{x+h}}{h}$ (B) $\lim\limits_{h\to 0}\frac{e^{x+h}-e^e}{h}$ (C) $\lim\limits_{h\to 0}\frac{e^{e+h}-e}{h}$

(D) $\lim\limits_{h\to 0}\frac{e^{x+h}-1}{h}$ (E) $\lim\limits_{h\to 0}\frac{e^{e+h}-e^e}{h}$

so... limit definition of derivative EVALUATED

when $x=e$

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$ use $f(x)=e^x$

$$\lim_{\Delta x\to 0}\frac{e^{x+\Delta x}-e^x}{\Delta x}$$ evaluated when $x=e$

so... $$\lim_{\Delta x\to 0}\frac{e^{e+\Delta x}-e^e}{\Delta x}$$