**CALCULUS AB**
**SECTION I, Part A**
**Time—55 minutes**
**Number of questions—28**

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

**Directions:** Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

**In this exam:**

(1) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function $f$ may be indicated using the inverse function notation $f^{-1}$ or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



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1. $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x^2 - 4}$ is

(A) $-\dfrac{1}{4}$ (B) 0 (C) 1 (D) $\dfrac{5}{4}$ (E) nonexistent

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2. If $f(x) = x^3 - x^2 + x - 1$, then $f'(2) =$

(A) 10 (B) 9 (C) 7 (D) 5 (E) 3



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3. Which of the following definite integrals has the same value as $\int_0^4 xe^{x^2}\,dx$ ?

(A) $\frac{1}{2}\int_0^4 e^u\,du$

(B) $\frac{1}{2}\int_0^{16} e^u\,du$

(C) $2\int_0^2 e^u\,du$

(D) $2\int_0^4 e^u\,du$

(E) $2\int_0^{16} e^u\,du$

4. Which of the following is an equation of the line tangent to the graph of $x^2 - 3xy = 10$ at the point $(1, -3)$ ?

(A) $y + 3 = -11(x - 1)$

(B) $y + 3 = -\frac{7}{3}(x - 1)$

(C) $y + 3 = \frac{1}{3}(x - 1)$

(D) $y + 3 = \frac{7}{3}(x - 1)$

(E) $y + 3 = \frac{11}{3}(x - 1)$



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5. If $g$ is the function given by $g(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 70x + 5$, on which of the following intervals is $g$ decreasing?

(A) $(-\infty, -10)$ and $(7, \infty)$

(B) $(-\infty, -7)$ and $(10, \infty)$

(C) $(-\infty, 10)$

(D) $(-10, 7)$

(E) $(-7, 10)$

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6. $\int_2^4 \frac{dx}{5 - 3x} =$

(A) $-\ln 7$ (B) $-\frac{\ln 7}{3}$ (C) $\frac{\ln 7}{3}$ (D) $\ln 7$ (E) $3\ln 7$



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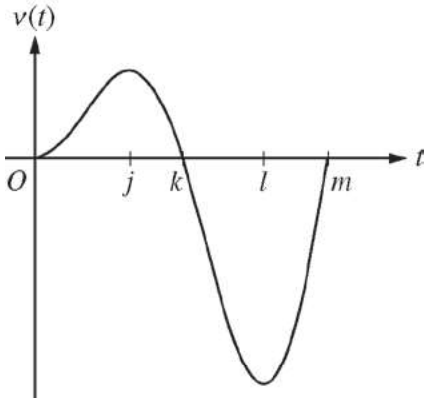
7. Let $f$ be the function given by $f(x) = x^3 - 6x^2 + 8x - 2$. What is the instantaneous rate of change of $f$ at $x = 3$?

(A) $-5$ (B) $-\frac{15}{4}$ (C) $-1$ (D) 6 (E) 17




8. A particle moves along a straight line. The graph of the particle's velocity $v(t)$ at time $t$ is shown above for $0 \le t \le m$, where $j$, $k$, $l$, and $m$ are constants. The graph intersects the horizontal axis at $t = 0$, $t = k$, and $t = m$ and has horizontal tangents at $t = j$ and $t = l$. For what values of $t$ is the speed of the particle decreasing?

(A) $j \le t \le l$

(B) $k \le t \le m$

(C) $j \le t \le k$ and $l \le t \le m$

(D) $0 \le t \le j$ and $k \le t \le l$

(E) $0 \le t \le j$ and $l \le t \le m$



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9. Let $f$ be the function given by $f(x) = \dfrac{(x-2)^2(x+3)}{(x-2)(x+1)}$. For which of the following values of $x$ is $f$ not continuous?

(A) $-3$ and $-1$ only

(B) $-3$, $-1$, and 2

(C) $-1$ only

(D) $-1$ and 2 only

(E) 2 only

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10. A particle moves along the $x$-axis with velocity given by $v(t) = 3t^2 - 4$ for time $t \geq 0$. If the particle is at position $x = -2$ at time $t = 0$, what is the position of the particle at time $t = 3$ ?

(A) 13 (B) 15 (C) 16 (D) 17 (E) 25



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11. Let $f$ be the function defined by $f(x) = \int_0^x \left(2t^3 - 15t^2 + 36t\right) dt$. On which of the following intervals is the graph of $y = f(x)$ concave down?

(A) $(-\infty, 0)$ only

(B) $(-\infty, 2)$

(C) $(0, \infty)$

(D) $(2, 3)$ only

(E) $(3, \infty)$ only

12. For which of the following does $\lim_{x \to \infty} f(x) = 0$?

I. $f(x) = \dfrac{\ln x}{x^{99}}$

II. $f(x) = \dfrac{e^x}{\ln x}$

III. $f(x) = \dfrac{x^{99}}{e^x}$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only



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13. Let $f$ be a differentiable function such that $f(0) = -5$ and $f'(x) \le 3$ for all $x$. Of the following, which is not a possible value for $f(2)$?

(A) $-10$ (B) $-5$ (C) 0 (D) 1 (E) 2

---

$$f(x) = \begin{cases} x + b & \text{if } x \le 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

14. Let $f$ be the function given above. What are all values of $a$ and $b$ for which $f$ is differentiable at $x = 1$?

(A) $a = \frac{1}{2}$ and $b = -\frac{1}{2}$

(B) $a = \frac{1}{2}$ and $b = \frac{3}{2}$

(C) $a = \frac{1}{2}$ and $b$ is any real number

(D) $a = b + 1$, where $b$ is any real number

(E) There are no such values of $a$ and $b$.



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| $f(3)$ | $g(3)$ | $f'(3)$ | $g'(3)$ |
|---|---|---|---|
| $-1$ | 2 | 5 | $-2$ |

15. The table above gives values for the functions $f$ and $g$ and their derivatives at $x = 3$. Let $k$ be the function given by $k(x) = \dfrac{f(x)}{g(x)}$, where $g(x) \neq 0$. What is the value of $k'(3)$ ?

(A) $-\dfrac{5}{2}$ (B) $-2$ (C) 2 (D) 3 (E) 8

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16. If $y = 5x\sqrt{x^2 + 1}$, then $\dfrac{dy}{dx}$ at $x = 3$ is

(A) $\dfrac{5}{2\sqrt{10}}$ (B) $\dfrac{15}{\sqrt{10}}$ (C) $\dfrac{15}{2\sqrt{10}} + 5\sqrt{10}$ (D) $\dfrac{45}{\sqrt{10}} + 5\sqrt{10}$ (E) $\dfrac{45}{\sqrt{10}} + 15\sqrt{10}$



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17. If $\lim_{h \to 0} \dfrac{\arcsin(a+h) - \arcsin(a)}{h} = 2$, which of the following could be the value of $a$ ?

(A) $\dfrac{\sqrt{2}}{2}$ (B) $\dfrac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\dfrac{1}{2}$ (E) 2

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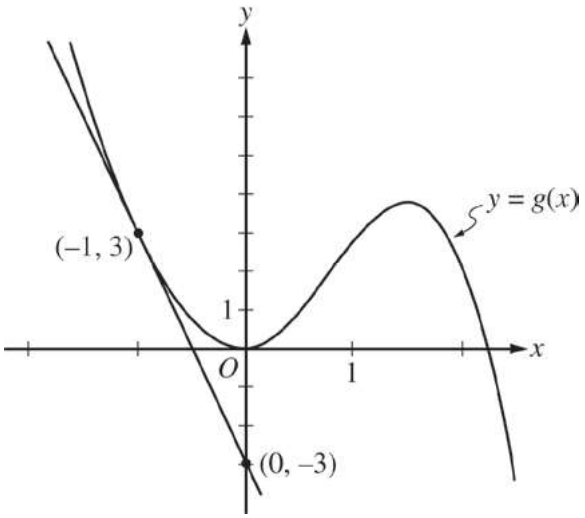
18. If $\ln(2x + y) = x + 1$, then $\dfrac{dy}{dx} =$

(A) $-2$ (B) $2x + y - 2$ (C) $2x + y$ (D) $4x + 2y - 2$ (E) $y - \dfrac{y}{x}$




19. The figure above shows the graph of the function $g$ and the line tangent to the graph of $g$ at $x = -1$. Let $h$ be the function given by $h(x) = e^x \cdot g(x)$. What is the value of $h'(-1)$ ?

(A) $\frac{9}{e}$ (B) $\frac{-3}{e}$ (C) $\frac{-6}{e}$ (D) $\frac{-6}{e} - \frac{3}{e^2}$ (E) $-6$

20. For $x > 0$, $\frac{d}{dx}\left(\int_0^{2x} \ln\left(t^3 + 1\right) dt\right) =$

(A) $\ln\left(x^3 + 1\right)$

(B) $\ln\left(8x^3 + 1\right)$

(C) $2\ln\left(x^3 + 1\right)$

(D) $2\ln\left(8x^3 + 1\right)$

(E) $24x^2 \ln\left(8x^3 + 1\right)$



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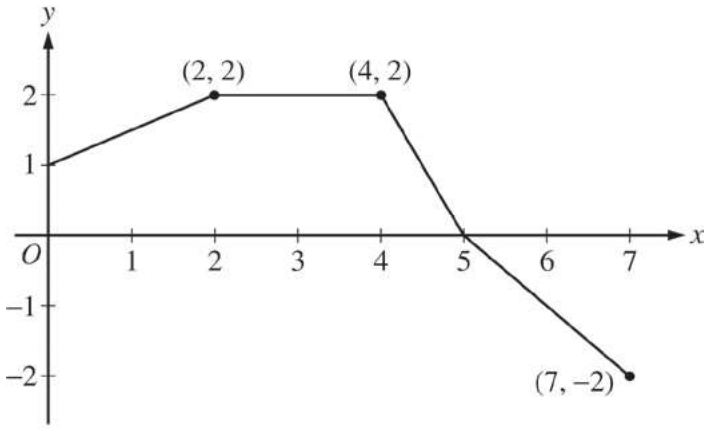


Graph of $f$

21. The graph of a function $f$ is shown above. What is the value of $\int_0^7 f(x)\,dx$ ?

(A) 6 (B) 8 (C) 10 (D) 14 (E) 18



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22. The function $f$ is continuous for all real numbers, and the average rate of change of $f$ on the closed interval $[6, 9]$ is $-\frac{3}{2}$. For $6 < c < 9$, there is no value of $c$ such that $f'(c) = -\frac{3}{2}$. Of the following, which must be true?

(A) $\frac{1}{3}\int_6^9 f(x)\,dx = -\frac{3}{2}$

(B) $\int_6^9 f(x)\,dx$ does not exist.

(C) $\frac{f'(6) + f'(9)}{2} = -\frac{3}{2}$

(D) $f'(x) < 0$ for all $x$ in the open interval $(6, 9)$.

(E) $f$ is not differentiable on the open interval $(6, 9)$.

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23. Let $f$ be the function defined by $f(x) = 2x + e^x$. If $g(x) = f^{-1}(x)$ for all $x$ and the point $(0, 1)$ is on the graph of $f$, what is the value of $g'(1)$ ?

(A) $\frac{1}{2 + e}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 3 (E) $2 + e$



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24. The function $g$ is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of $g$ on the closed interval $[-2, 1]$?

(A) $-7$ (B) $-\frac{3}{4}$ (C) 0 (D) 2 (E) 6

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25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln\left(-e^x + 5\right)$

(D) $y = -\ln\left(e^x + 3\right)$

(E) $y = \ln\left(e^x + 3\right)$



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26. Which of the following is an antiderivative of $f(x) = \sqrt{1 + x^3}$ ?

(A) $\frac{2}{3}\left(1 + x^3\right)^{3/2}$

(B) $\frac{\frac{2}{3}\left(1 + x^3\right)^{3/2}}{3x^2}$

(C) $\int_0^{1+x^3} \sqrt{t}\ dt$

(D) $\int_0^{x^3} \sqrt{1 + t}\ dt$

(E) $\int_0^{x} \sqrt{1 + t^3}\ dt$

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27. For time $t \geq 0$, the height $h$ of an object suspended from a spring is given by $h(t) = 16 + 7\cos\left(\frac{\pi t}{4}\right)$. What is the average height of the object from $t = 0$ to $t = 2$ ?

(A) 16 (B) $\frac{39}{2}$ (C) $16 - \frac{14}{\pi}$ (D) $16 + \frac{14}{\pi}$ (E) $32 + \frac{28}{\pi}$



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28. The function $f$ is defined by $f(x) = \sin x + \cos x$ for $0 \le x \le 2\pi$. What is the $x$-coordinate of the point of inflection where the graph of $f$ changes from concave down to concave up?

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$ (E) $\frac{9\pi}{4}$

**END OF PART A OF SECTION I**

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.**

**DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.**

**CALCULUS AB**
**SECTION I, Part B**
**Time—50 minutes**
**Number of questions—17**

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

**Directions:** Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

**BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.**

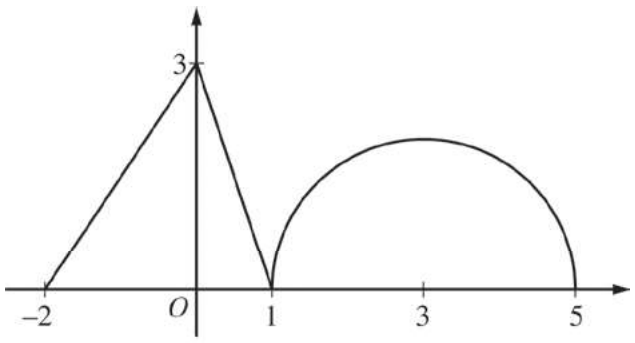
**YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.**

**In this exam:**

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function $f$ may be indicated using the inverse function notation $f^{-1}$ or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).




Graph of $f$

76. The graph of the function $f$ shown above consists of two line segments and a semicircle. Let $g$ be defined by $g(x) = \int_0^x f(t)\,dt$. What is the value of $g(5)$?

(A) 0 (B) $-1.5 + 2\pi$ (C) $2\pi$ (D) $1.5 + 2\pi$ (E) $4.5 + 2\pi$



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77. The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeter per second, what is the radius of the sphere?

(The volume $V$ of a sphere with radius $r$ is $V = \frac{4}{3}\pi r^3$.)

(A) 0.141 cm (B) 0.244 cm (C) 0.250 cm (D) 0.489 cm (E) 0.977 cm

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78. Let $f$ and $g$ be continuous functions such that $\int_0^{10} f(x)\,dx = 21$, $\int_0^{10} \frac{1}{2}g(x)\,dx = 8$, and $\int_3^{10} (f(x) - g(x))\,dx = 2$. What is the value of $\int_0^3 (f(x) - g(x))\,dx$ ?

(A) 3 (B) 7 (C) 11 (D) 15 (E) 19



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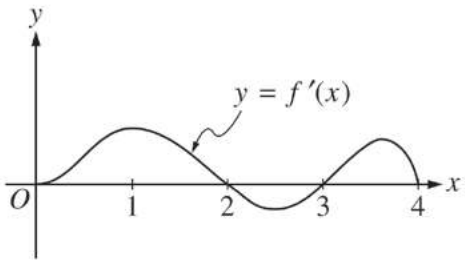
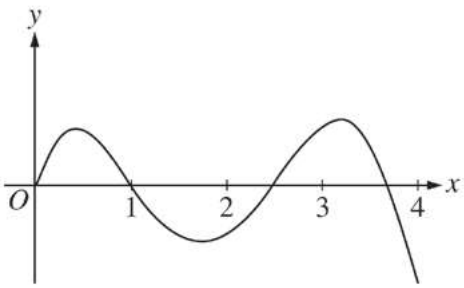
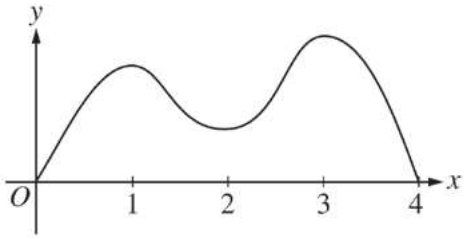



79. The figure above shows the graph of $f'$, the derivative of the function $f$. If $f(0) = 0$, which of the following could be the graph of $f$?

(A)




(B)

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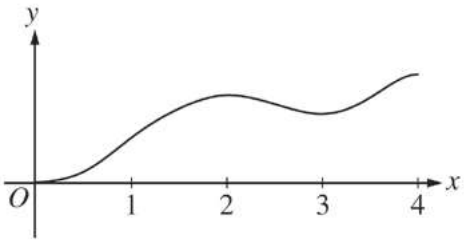


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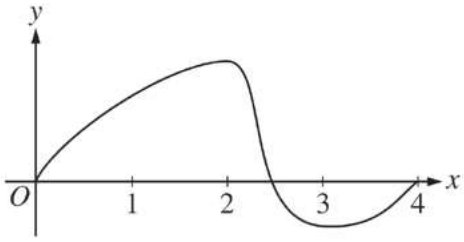
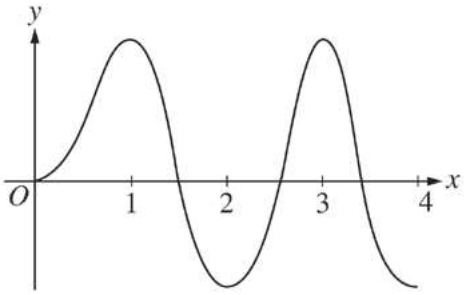



(E)



80. For time $t \geq 0$, the position of a particle traveling along a line is given by a differentiable function $s$. If $s$ is increasing for $0 \leq t < 2$ and $s$ is decreasing for $t > 2$, which of the following is the total distance the particle travels for $0 \leq t \leq 5$ ?

(A) $s(0) + \int_0^2 s'(t)\,dt - \int_2^5 s'(t)\,dt$

(B) $s(0) + \int_2^5 s'(t)\,dt - \int_0^2 s'(t)\,dt$

(C) $\int_2^5 s'(t)\,dt - \int_0^2 s'(t)\,dt$

(D) $\left|\int_0^5 s'(t)\,dt\right|$

(E) $\int_0^5 |s'(t)|\,dt$



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81. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit $(°F)$. If the initial temperature of the tea, at time $t = 0$ minutes, is $200°F$ and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) 175°F (B) 130°F (C) 95°F (D) 70°F (E) 45°F

82. The derivative of the function $f$ is given by $f'(x) = x^3 - 4\sin\left(x^2\right) + 1$. On the interval $(-2.5, 2.5)$, at which of the following values of $x$ does $f$ have a relative maximum?

(A) $-1.970$ and 0

(B) $-1.467$ and 1.075

(C) $-0.475$, 0.542, and 1.396

(D) $-0.475$ and 1.396 only

(E) 0.542 only

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | 4 | 10 | 18 | 28 | 40 | 54 |

83. The table above gives selected values for a continuous function $f$. If $f$ is increasing over the closed interval $[0, 3]$, which of the following could be the value of $\int_0^3 f(x)\,dx$?

(A) 50 (B) 62 (C) 77 (D) 100 (E) 154



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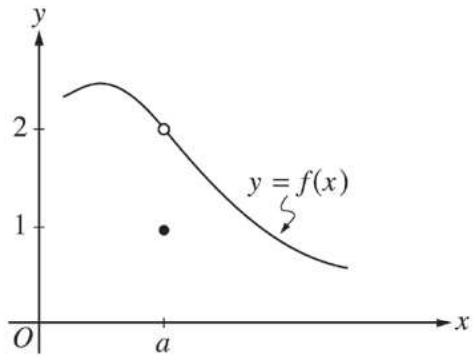


84. The graph of a function $f$ is shown in the figure above. Which of the following statements is true?

(A) $f(a) = 2$

(B) $f$ is continuous at $x = a$.

(C) $\lim_{x \to a} f(x) = 1$

(D) $\lim_{x \to a} f(x) = 2$

(E) $\lim_{x \to a} f(x)$ does not exist.

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85. A particle moves along the $x$-axis so that at time $t \geq 0$ its position is given by $x(t) = \cos \sqrt{t}$. What is the velocity of the particle at the first instance the particle is at the origin?

(A) $-1$ (B) $-0.624$ (C) $-0.318$ (D) $0$ (E) $0.065$



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86. If $f'(x) > 0$ for all $x$ and $f''(x) < 0$ for all $x$, which of the following could be a table of values for $f$?

(A)

| $x$ | $f(x)$ |
|---|---|
| –1 | 4 |
| 0 | 3 |
| 1 | 1 |

(B)

| $x$ | $f(x)$ |
|---|---|
| –1 | 4 |
| 0 | 4 |
| 1 | 4 |

(C)

| $x$ | $f(x)$ |
|---|---|
| –1 | 4 |
| 0 | 5 |
| 1 | 6 |

(D)

| $x$ | $f(x)$ |
|---|---|
| –1 | 4 |
| 0 | 5 |
| 1 | 7 |

(E)

| $x$ | $f(x)$ |
|---|---|
| –1 | 4 |
| 0 | 6 |
| 1 | 7 |



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87. Let $f$ be the function with first derivative given by $f'(x) = \left(3 - 2x - x^2\right)\sin(2x - 3)$. How many relative extrema does $f$ have on the open interval $-4 < x < 2$?

(A) Two (B) Three (C) Four (D) Five (E) Six



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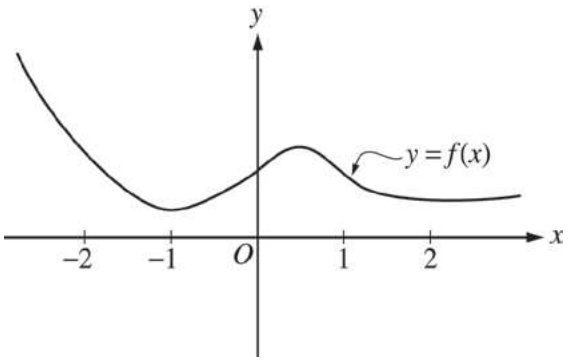


88. The graph of a twice-differentiable function $f$ is shown in the figure above. Which of the following is true?

(A) $f'(-1) < f'(1) < f'(0)$

(B) $f'(-1) < f'(0) < f'(1)$

(C) $f'(0) < f'(-1) < f'(1)$

(D) $f'(1) < f'(-1) < f'(0)$

(E) $f'(1) < f'(0) < f'(-1)$



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89. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y-2}$ and the lines $x = 0$ and $y = 5$ is revolved about the $y$-axis?

(A) 3.464 (B) 4.500 (C) 7.854 (D) 10.883 (E) 14.137

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90. The population $P$ of a city grows according to the differential equation $\frac{dP}{dt} = kP$, where $k$ is a constant and $t$ is measured in years. If the population of the city doubles every 12 years, what is the value of $k$ ?

(A) 0.058 (B) 0.061 (C) 0.167 (D) 0.693 (E) 8.318



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91. The function $f$ is continuous and $\int_0^8 f(u)\,du = 6$. What is the value of $\int_1^3 xf\left(x^2 - 1\right)\,dx$ ?

(A) $\frac{3}{2}$ (B) 3 (C) 6 (D) 12 (E) 24

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92. The function $f$ is defined for all $x$ in the closed interval $[a, b]$. If $f$ does not attain a maximum value on $[a, b]$, which of the following must be true?

(A) $f$ is not continuous on $[a, b]$.

(B) $f$ is not bounded on $[a, b]$.

(C) $f$ does not attain a minimum value on $[a, b]$.

(D) The graph of $f$ has a vertical asymptote in the interval $[a, b]$.

(E) The equation $f'(x) = 0$ does not have a solution in the interval $[a, b]$.



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