CollegeBoard

Calculus AB
Practice Exam and Notes

Important Note

This Practice Exam is provided by the College Board for AP Exam preparation. Teachers are permitted to download the materials and make copies to use with their students in a classroom setting only. To maintain the security of this exam, teachers should collect all materials after their administration and keep them in a secure location.

Exams may not be posted on school or personal websites, nor electronically redistributed for any reason. Further distribution of these materials outside of the secure College Board site disadvantages teachers who rely on uncirculated questions for classroom testing. Any additional distribution is in violation of the College Board's copyright policies and may result in the termination of Practice Exam access for your school as well as the removal of access to other online services such as the AP Teacher Community and Online Score Reports.

Effective Fall 2016

About the College Board

The College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, the College Board was created to expand access to higher education. Today, the membership association is made up of over 6,000 of the world's leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, the College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success — including the SAT® and the Advanced Placement Program®. The organization also serves the education community through research and advocacy on behalf of students, educators, and schools. For further information, visit **www.collegeboard.org**.

AP® Equity and Access Policy

The College Board strongly encourages educators to make equitable access a guiding principle for their AP® programs by giving all willing and academically prepared students the opportunity to participate in AP. We encourage the elimination of barriers that restrict access to AP for students from ethnic, racial and socioeconomic groups that have been traditionally underrepresented. Schools should make every effort to ensure their AP classes reflect the diversity of their student population. The College Board also believes that all students should have access to academically challenging course work before they enroll in AP classes, which can prepare them for AP success. It is only through a commitment to equitable preparation and access that true equity and excellence can be achieved.

Contents

Introduction

Beginning in May 2017, the AP Calculus AB Exam will measure students' ability to apply strategies and techniques to accurately solve diverse types of problems. The revised exam will feature the same number of questions and total allotted time, but the distribution of questions and relative timing have been adjusted based on feedback from teachers and administrators, and multiple-choice questions now have four answer choices instead of five.

Part I of this publication is the AP Calculus AB Practice Exam. This will mirror the look and feel of an actual AP Exam, including instructions and sample questions. However, these exam questions have never been administered as an operational exam, and, therefore, statistical analysis is **not** available. The purpose of this section is to provide educators with sample exam questions that accurately reflect the composition/design of the revised exam and to offer these questions in a way that gives teachers the opportunity to test their students in an exam situation that closely resembles the actual exam administration.

Important: Final instructions for every AP Exam are published in the *AP Exam Instructions* book. Please reference that publication, which is posted at **www.collegeboard.org/apexaminstructions** in March and included in schools' exam shipments, for the final instructions and format of this AP Exam.

Part II is the Notes on the AP Calculus AB Practice Exam. This section offers detailed explanations of how each question in the practice exam links back to the curriculum framework in order to provide a clear link between curriculum and assessment. The multiple-choice rationales explain the correct answer and incorrect options. Scoring information is provided for the free-response section.

How AP Courses and Exams Are Developed

AP courses and exams are designed by committees of college faculty and AP teachers who ensure that each AP course and exam reflects and assesses college-level expectations. These committees define the scope and expectations of the course, articulating through a curriculum framework what students should know and be able to do upon completion of the AP course. Their work is informed by data collected from a range of colleges and universities to ensure that AP course work reflects current scholarship and advances in the discipline.

These same committees are also responsible for designing and approving exam specifications and exam questions that clearly connect to the curriculum framework. The AP Exam development process is a multiyear endeavor; all AP Exams undergo extensive review, revision, piloting, and analysis to ensure that questions are high quality and fair and that the questions comprise an appropriate range of difficulty.

Throughout AP course and exam development, the College Board gathers feedback from secondary and postsecondary educators. This feedback is carefully considered to ensure that AP courses and exams provide students with a college-level learning experience and the opportunity to demonstrate their qualifications for advanced placement and college credit upon college entrance.

Methodology Guiding the Revision

The course and the exam are conceived and developed using similar methodologies. The course is designed using the principles of *Understanding by Design*, and the exam is designed and developed using the similarly principled evidence-centered design approach. Both processes begin by identifying the end goals that identify what students should know and be able to do by the end of their AP experience. These statements about students' knowledge and abilities, along with descriptions of the observable evidence that delineate levels of student performance, serve simultaneously as the learning objectives for the course and the targets of measurement for the exam. The course and exam, by design, share the same foundation.

Course Development

Each committee first articulates its discipline's high-level goals before identifying the course's specific learning objectives. This approach is consistent with "backward design" — the practice of developing curricula, instruction, and assessments with the end goal in mind. The learning objectives describe what students should know and be able to do, thereby providing clear instructional goals as well as targets of measurement for the exam.

Exam Development

Exam development begins with the committee making decisions about the overall nature of the exam. How will the learning objectives for the course be assessed? How will the course content and skills be distributed across the exam? How many multiple-choice questions should there be? How many free-response questions should be included? How much time will be devoted to each section? Answers to these questions become part of the exam specifications.

With the exam specifications set, assessment specialists design questions that conform to these specifications. The committee reviews every exam question for alignment with the curriculum framework, accuracy, and a number of other criteria that ensure the integrity of the exam.

Exam questions are then piloted in AP classrooms to determine their statistical properties. Questions that have been approved by the committee and piloted successfully are included in an exam. When an exam is assembled, the committee conducts a final review to ensure overall conformity with the specifications.

How AP Exams Are Scored

The exam scoring process, like the course and exam development process, relies on the expertise of both AP teachers and college faculty. While multiple-choice questions are scored by machine, the free-response questions and, as applicable, through-course performance assessments, are scored by college faculty and expert AP teachers at the annual AP Reading. Most of the Reading occurs in face-to-face settings, while a small portion are scored online while the face-to-face Reading is taking place.

AP Exam Readers are thoroughly trained, and their work is monitored throughout the Reading for fairness and consistency. In each subject, a highly respected college faculty member fills the role of Chief Reader, who, with the help of AP Readers in leadership positions, maintains the accuracy of the scoring standards. Scores on the free-response questions and performance assessments are weighted and combined with the weighted results of the computer-scored multiple-choice questions to yield the weighted composite score, and this composite score is converted into an AP exam score of 5, 4, 3, 2, or 1.

The score-setting process is both precise and labor intensive, involving numerous psychometric analyses of the results of a specific AP Exam in a specific year and of the particular group of students who took that exam. Additionally, to ensure alignment with college-level standards, part of the score-setting process involves comparing the performance of AP students with the performance of students enrolled in comparable courses in colleges throughout the United States. In general, the AP composite score points are set so that the lowest raw score needed to earn an AP score of 5 is equivalent to the average score among college students earning grades of A in the college course. Similarly, AP Exam scores of 4 are equivalent to college grades of A–, B+, and B. AP Exam scores of 3 are equivalent to college grades of B–, C+, and C.

Using and Interpreting AP Scores

The extensive work done by college faculty and AP teachers in the development of the course and the exam and throughout the scoring process ensures that AP Exam scores accurately represent students' achievement in the equivalent college course. While colleges and universities are responsible for setting their own credit and placement policies, AP scores signify how qualified students are to receive college credit and placement:

AP Score	Recommendation
5	Extremely well qualified
4	Well qualified
3	Qualified
2	Possibly qualified
1	No recommendation

Additional Resources

Visit **apcentral.collegeboard.org** for more information about the AP Program.

AP Calculus AB Practice Exam

Exam Content and Format

The 2017 AP Calculus AB Exam is 3 hours and 15 minutes in length. There are two sections:

- Section I is 1 hour, 45 minutes and consists of 45 multiple-choice questions in two separately-timed parts, accounting for 50 percent of the final score. Part A consists of 30 questions in 60 minutes and does not allow the use of a calculator. Part B consists of 15 questions in 45 minutes and requires the use of a graphing calculator.
- Section II is 1 hour, 30 minutes and consists of 6 free-response questions in two separately-timed parts, accounting for 50 percent of the final score. Part A consists of 2 questions in 30 minutes and requires the use of a graphing calculator. Part B consists of 4 questions in 60 minutes and does not allow the use of a calculator. During the timed portion for Part B, students are permitted to continue to work on questions in Part A, but they are not allowed to use a calculator during this time.

Administering the Practice Exam

This section contains instructions for administering the AP Calculus AB Practice Exam. You may wish to use these instructions to create an exam situation that resembles an actual administration. If so, read the indented, boldface directions to the students; all other instructions are for administering the exam and need not be read aloud. Before beginning testing, have all exam materials ready for distribution. These include exam booklets and answer sheets.

Graphing calculators are required to answer some of the questions on the AP Calculus Exams. Before starting the exam administration, make sure each student has a graphing calculator from the approved list at **https://apstudent.collegeboard.org/apcourse/ap-calculus-ab/calculator-policy** or on AP Central. During the administration of Section I, Part B, and Section II, Part A, students may have no more than two graphing calculators on their desks. Calculators may not be shared. Calculator memories do not need to be cleared before or after the exam. Since graphing calculators can be used to store data, including text, teachers should monitor that students are using their calculators appropriately.

Section I of the Practice Exam should be completed using a No. 2 pencil to simulate an actual administration. Students may use a No. 2 pencil or a pen with black or dark blue ink to complete Section II.

Instructions for the Section II free-response questions are included in this publication. It is important to share these with students and ask them to read these instructions carefully at the beginning of the administration of Section II. Timing for Section II should begin after you have given sufficient time to read these instructions.

SECTION I, Multiple-Choice Questions — Part A (no calculator allowed) and Part B (graphing calculator required)

When you are ready to begin Section I, say:

> **Section I is the multiple-choice portion of the exam. Mark all of your responses on your answer sheet, one response per question. If you need to erase, do so carefully and completely. Your score on the multiple-choice section will be based solely on the number of questions answered correctly.**
>
> **Section I is divided into two parts. Each part is timed separately, and you may work on each part only during the time allotted for it. Calculators are not allowed in Part A. Please put all of your calculators under your chair. Are there any questions?…**
>
> **You have 60 minutes for Part A. Part A questions are numbered 1 through 30. Open your Section I booklet to Part A and begin.**

Note Start Time here_____ . Note Stop Time here_____ . Check that students are marking their answers in pencil on the answer sheets and that they are not looking beyond Part A. The line of A's at the top of each page will assist you in monitoring students' work. After 50 minutes, say:

> **There are 10 minutes remaining in Part A.**

After 10 minutes, say:

> **Stop working on Part A. Graphing calculators are required for Part B. You may get your calculators from under your chair and place them on your desk. You have 45 minutes for Part B. Part B questions are numbered 76 through 90. Open your Section I booklet to Part B and begin.**

Note Start Time here_____ . Note Stop Time here_____ . Check that students are marking their answers in pencil on the answer sheets and are now working on Part B. The large B's in an alternating shaded pattern at the top of each page will assist you in monitoring their work. After 35 minutes, say:

> **There are 10 minutes remaining in Part B.**

After 10 minutes, say:

> **Stop working and close your exam booklet. I will now collect your Section I booklet and your answer sheet. Put your exam booklet and your answer sheet on your desk, face up. Remain in your seat, without talking, while the exam materials are collected.**

Collect a Section I booklet and answer sheet from each student.

There is a 10-minute break between Sections I and II.

SECTION II: Free-Response Questions — Part A (graphing calculator required) and Part B (no calculator allowed)

After the break, say:

> **Section II is the free-response portion of the exam. It also has two parts that are timed separately. You are responsible for pacing yourself, and may proceed freely from one question to another within each part. Graphing calculators are required for Part A, so you may keep your calculators on your desk. You must write your answers in the appropriate space in the exam booklet using a No. 2 pencil or a pen with black or dark blue ink. Do not begin Part B until you are told to do so.**
>
> **Please read the instructions for the free-response section, paying careful attention to the bulleted statements. Look up when you have finished….**
>
> **Are there any questions?...**
>
> **You have 30 minutes to answer the questions in Part A. Open your Section II booklet and begin.**

Note Start Time here_____ . Note Stop Time here_____ . Check that students are working on Part A only and writing their answers in their exam booklets using pencils or pens with black or dark blue ink. The pages for Part A questions are marked with large 1s or 2s at the top of each page to assist you in monitoring their work. After 20 minutes, say:

> **There are 10 minutes remaining in Part A.**

After 10 minutes, say:

> **Stop working on Part A. Calculators are not allowed for Part B. Please put all of your calculators under your chair….**
>
> **You have 60 minutes for Part B. During this time you may go back to Part A, but you may not use your calculator. Remember to show your work, and write your answer to each part of each question in the appropriate space in the exam booklet. Are there any questions?...**
>
> **Open your Section II booklet to Part B and begin.**

Note Start Time here_____ . Note Stop Time here_____ . After 50 minutes, say:

> **There are 10 minutes remaining in Part B.**

After 10 minutes, say:

> **Stop working and close your exam booklet. Put your exam booklet on your desk, face up. Remain in your seat, without talking, while the exam materials are collected.**

Collect a Section II booklet from each student. Then say:

> **The exam is now completed.**

Name: _____

AP® Calculus AB
Answer Sheet
for Multiple-Choice Section

No.	Answer
1	
2	
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27	
28	
29	
30	

No.	Answer
76	
77	
78	
79	
80	
81	
82	
83	
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85	
86	
87	
88	
89	
90	

AP® Calculus AB Exam

SECTION I: Multiple Choice

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
30
Time
60 minutes
Electronic Device
None allowed

Part B

Number of Questions
15
Time
45 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions. For Part A, fill in only the boxes for numbers 1 through 30 on the answer sheet. For Part B, fill in only the boxes for numbers 76 through 90 on the answer sheet.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, place the letter of your choice in the corresponding box on the answer sheet. Give only one answer to each question.

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A

CALCULUS AB

SECTION I, Part A

Time—60 minutes

Number of questions—30

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and place the letter of your choice in the corresponding box on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. $\int_{-1}^{3} (x^2 - 2x)\, dx =$

(A) $-\frac{2}{3}$ (B) $\frac{4}{3}$ (C) 8 (D) 12

---

2. If $f(x) = e^{2x}(x^3 + 1)$, then $f'(2) =$

(A) $6e^4$ (B) $21e^4$ (C) $24e^4$ (D) $30e^4$

GO ON TO THE NEXT PAGE.

3. Let f be a differentiable function such that $f(2) = 4$ and $f'(2) = -\frac{1}{2}$. What is the approximation for $f(2.1)$ found by using the line tangent to the graph of f at $x = 2$?

   (A) 2.95 (B) 3.95 (C) 4.05 (D) 4.1

---

4. Let g be the function defined by $g(x) = x^4 + 4x^3$. How many relative extrema does g have?

   (A) Zero (B) One (C) Two (D) Three

GO ON TO THE NEXT PAGE.

5. The velocity of a particle moving along the x-axis is given by $v(t) = 2 - t^2$ for time $t > 0$. What is the average velocity of the particle from time $t = 1$ to time $t = 3$?

   (A) -4 (B) -3 (C) $-\frac{7}{3}$ (D) $\frac{7}{3}$

t (hours)	0	2	7	9
$R(t)$ (tons per hour)	15	9	5	4

6. On a certain day, the rate at which material is deposited at a recycling center is modeled by the function R, where $R(t)$ is measured in tons per hour and t is the number of hours since the center opened. Using a trapezoidal sum with the three subintervals indicated by the data in the table, what is the approximate number of tons of material deposited in the first 9 hours since the center opened?

   (A) 68 (B) 70.5 (C) 85 (D) 136

GO ON TO THE NEXT PAGE.

7. What is the total area of the regions between the curves $y = 6x^2 - 18x$ and $y = -6x$ from $x = 1$ to $x = 3$?

   (A) 4 (B) 12 (C) 16 (D) 20

---

8. The function g is defined by $g(x) = x^2 + bx$, where b is a constant. If the line tangent to the graph of g at $x = -1$ is parallel to the line that contains the points $(0, -2)$ and $(3, 4)$, what is the value of b ?

   (A) -1 (B) 2 (C) $\frac{5}{2}$ (D) 4

GO ON TO THE NEXT PAGE.

$$f(x) = \begin{cases} \dfrac{|x|}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

9. The function f is defined above. The value of $\int_{-5}^{3} f(x)\,dx$ is

   (A) -2 (B) 2 (C) 8 (D) nonexistent

---

10. Let g be a continuous function. Using the substitution $u = 2x - 1$, the integral $\int_{2}^{3} g(2x - 1)\,dx$ is equal to which of the following?

    (A) $\int_{2}^{3} g(u)\,du$ (B) $\frac{1}{2}\int_{2}^{3} g(u)\,du$ (C) $\int_{3}^{5} g(u)\,du$ (D) $\frac{1}{2}\int_{3}^{5} g(u)\,du$

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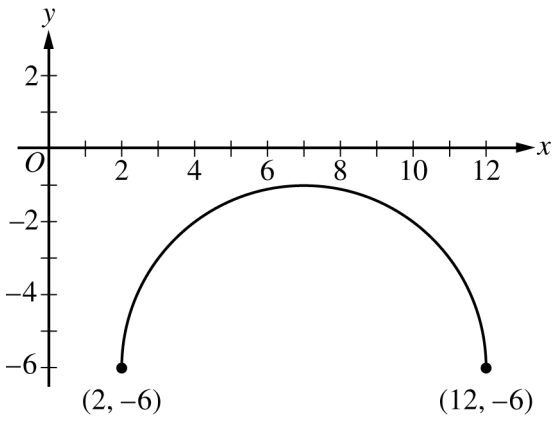


Graph of f

11. The graph of $y = f(x)$ consists of a semicircle with endpoints at $(2, -6)$ and $(12, -6)$, as shown in the figure above. What is the value of $\int_2^{12} f(x)\,dx$?

(A) $-\frac{25\pi}{2}$ (B) $\frac{25\pi}{2}$ (C) $-60 + \frac{25\pi}{2}$ (D) $60 - \frac{25\pi}{2}$

GO ON TO THE NEXT PAGE.

12. An object moves along a straight line so that at any time t its acceleration is given by $a(t) = 6t$. At time $t = 0$, the object's velocity is 10 and the object's position is 7. What is the object's position at time $t = 2$?

(A) 22 (B) 27 (C) 28 (D) 35

---

13. If $y = \cos x - \ln(2x)$, then $\frac{d^3y}{dx^3} =$

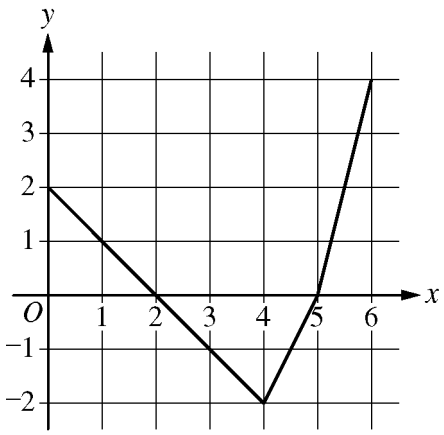
(A) $\sin x - \frac{2}{x^3}$

(B) $-\sin x - \frac{2}{x^3}$

(C) $\sin x - \frac{1}{x^3}$

(D) $-\sin x - \frac{1}{x^3}$

GO ON TO THE NEXT PAGE.



Graph of f

14. The graph of the function f, shown above, consists of three line segments. If the function g is an antiderivative of f such that $g(2) = 5$, for how many values of c, where $0 \le c \le 6$, does $g(c) = 3$?

(A) Zero (B) One (C) Two (D) Three

$$f(x) = \begin{cases} 6 + cx & \text{for } x < 1 \\ 9 + 2\ln x & \text{for } x \ge 1 \end{cases}$$

15. Let f be the function defined above, where c is a constant. If f is continuous at $x = 1$, what is the value of c ?

(A) 2 (B) 3 (C) 5 (D) 9

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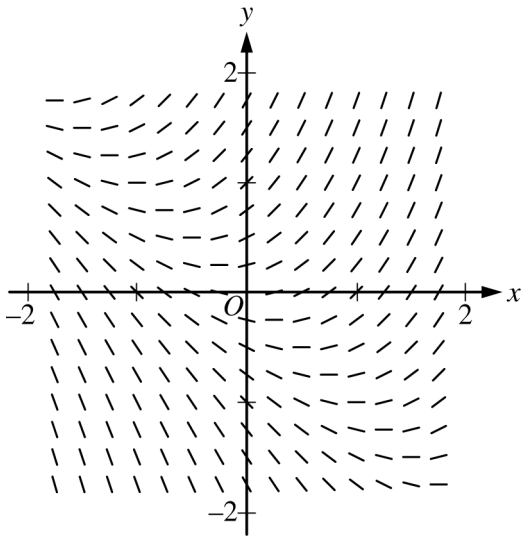
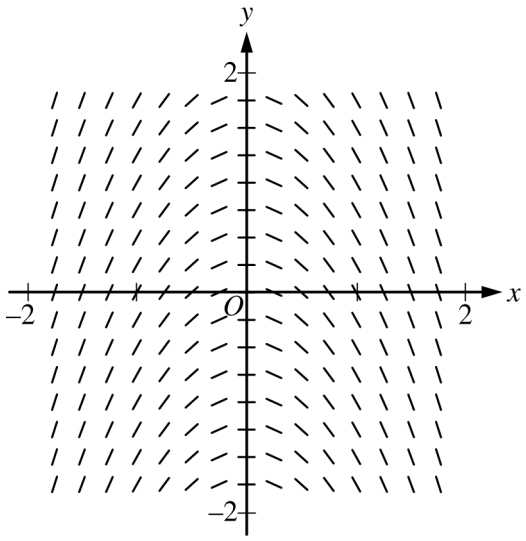
16. Which of the following could be a slope field for the differential equation $\frac{dy}{dx} = x^2 + y$?

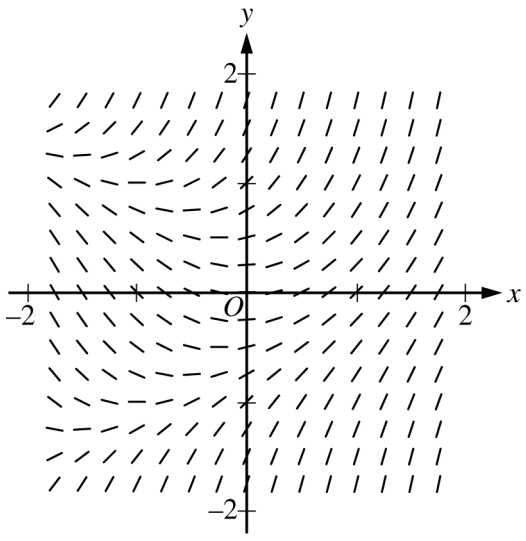
(A)



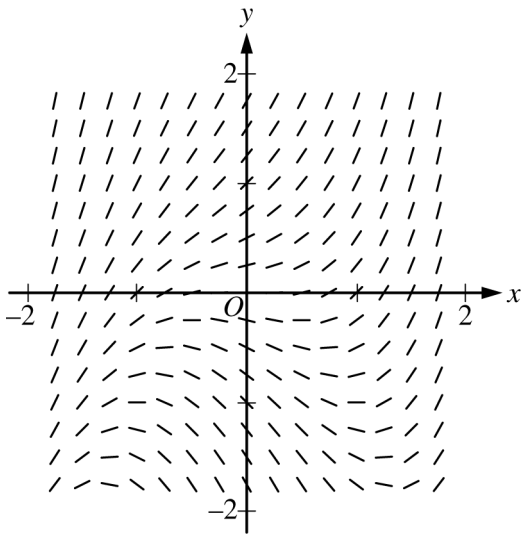
(B)



(C)



(D)



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17. The function $y = e^{3x} - 5x + 7$ is a solution to which of the following differential equations?

(A) $y'' - 3y' - 15 = 0$

(B) $y'' - 3y' + 15 = 0$

(C) $y'' - y' - 5 = 0$

(D) $y'' - y' + 5 = 0$

---

18. If $f(x) = \sin^{-1} x$, then $f'\left(\frac{\sqrt{3}}{2}\right) =$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{4}{7}$ (D) 2

GO ON TO THE NEXT PAGE.

19. $\lim_{x \to e} \dfrac{(x^{20} - 3x) - (e^{20} - 3e)}{x - e}$ is

(A) 0 (B) $20e^{19} - 3$ (C) $e^{20} - 3e$ (D) nonexistent

---

20. Let $y = f(x)$ be a twice-differentiable function such that $f(1) = 2$ and $\dfrac{dy}{dx} = y^3 + 3$. What is the value of $\dfrac{d^2y}{dx^2}$ at $x = 1$?

(A) 12 (B) 66 (C) 132 (D) 165

GO ON TO THE NEXT PAGE.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
–2	–6	9	–10	16
1	5	–3	3	–2
3	0	7	8	3

21. The table above gives values of f, f', g, and g' for selected values of x. If $h(x) = f(g(x))$, what is the value of $h'(1)$?

(A) -19 (B) -14 (C) 7 (D) 9

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22. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$ with the initial condition $f(0) = -2$. Which of the following is an expression for $f(x)$?

(A) $-2 - \sqrt{x^2 + 2x}$

(B) $-2 + \sqrt{x^2 + 2x}$

(C) $-\sqrt{x^2 + 2x + 4}$

(D) $\sqrt{x^2 + 2x + 4}$

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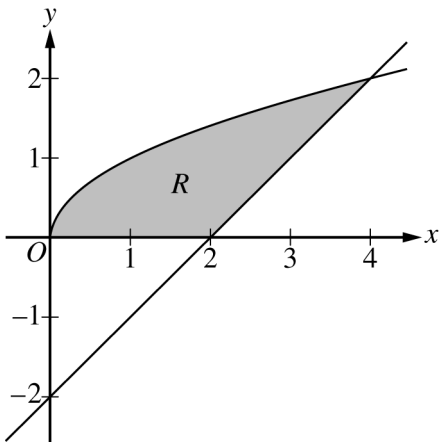


23. Let R be the shaded region bounded by the graph of $y = \sqrt{x}$, the graph of $y = x - 2$, and the x-axis, as shown in the figure above. Which of the following gives the volume of the solid generated when R is revolved about the x-axis?

(A) $\pi \int_0^4 \left(x - (x-2)^2\right) dx$

(B) $\pi \int_0^4 \left(\sqrt{x} - (x-2)\right)^2 dx$

(C) $\pi \int_0^2 x\, dx + \pi \int_2^4 \left(x - (x-2)^2\right) dx$

(D) $\pi \int_0^2 x\, dx + \pi \int_2^4 \left(\sqrt{x} - (x-2)\right)^2 dx$

GO ON TO THE NEXT PAGE.

24. $\lim_{x \to 3} \dfrac{\tan(x-3)}{3e^{x-3} - x}$ is

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) nonexistent

---

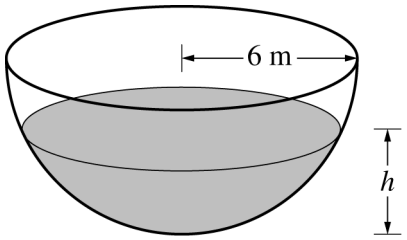
25. Let f be a function with first derivative defined by $f'(x) = \dfrac{3x^2 - 6}{x^2}$ for $x > 0$. It is known that $f(1) = 9$ and $f(3) = 11$. What value of x in the open interval $(1, 3)$ satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[1, 3]$?

(A) $\sqrt{6}$ (B) $\sqrt{3}$ (C) $\sqrt{2}$ (D) 1

GO ON TO THE NEXT PAGE.

26. $\displaystyle\int_1^2 \frac{x^2 - x - 5}{x + 2}\,dx =$

(A) $-\frac{3}{2} + \ln\frac{4}{3}$ (B) $-\frac{25}{21}$ (C) $\frac{5}{2} + 3\ln\frac{3}{4}$ (D) $\frac{23}{45}$



27. A hemispherical water tank, shown above, has a radius of 6 meters and is losing water. The area of the surface of the water is $A = 12\pi h - \pi h^2$ square meters, where h is the depth, in meters, of the water in the tank. When $h = 3$ meters, the depth of the water is decreasing at a rate of $\frac{1}{2}$ meter per minute. At that instant, what is the rate at which the area of the water's surface is decreasing with respect to time?

(A) 3π square meters per minute

(B) 6π square meters per minute

(C) 9π square meters per minute

(D) 27π square meters per minute

GO ON TO THE NEXT PAGE.

28. Consider a triangle in the xy-plane. Two vertices of the triangle are on the x-axis at $(1, 0)$ and $(5, 0)$, and a third vertex is on the graph of $y = \ln(2x) - \frac{1}{2}x + 5$ for $\frac{1}{2} \le x \le 8$. What is the maximum area of such a triangle?

(A) $\frac{19}{2}$

(B) $2 \ln 2 + 9$

(C) $2 \ln 4 + 8$

(D) $2 \ln 16 + 2$

---

29. The function f is defined by $f(x) = x^3 + 4x + 2$. If g is the inverse function of f and $g(2) = 0$, what is the value of $g'(2)$?

(A) $-\frac{1}{16}$ (B) $-\frac{4}{81}$ (C) $\frac{1}{4}$ (D) 4

GO ON TO THE NEXT PAGE.

30. Which of the following limits is equal to $\int_2^5 x^2\,dx$?

(A) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + \frac{k}{n}\right)^2 \frac{1}{n}$

(B) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + \frac{k}{n}\right)^2 \frac{3}{n}$

(C) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + \frac{3k}{n}\right)^2 \frac{1}{n}$

(D) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + \frac{3k}{n}\right)^2 \frac{3}{n}$

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION I, Part B

Time—45 minutes

Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and place the letter of your choice in the corresponding box on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

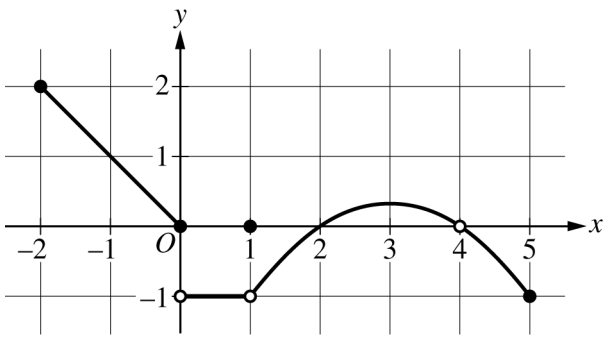
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

76. To help restore a beach, sand is being added to the beach at a rate of $s(t) = 65 + 24\sin(0.3t)$ tons per hour, where t is measured in hours since 5:00 A.M. How many tons of sand are added to the beach over the 3-hour period from 7:00 A.M. to 10:00 A.M.?

(A) 255.368 (B) 225.271 (C) 85.123 (D) 10.388



Graph of f

77. The graph of the function f is shown above. For what values of a does $\lim_{x \to a} f(x) = 0$?

(A) 2 only

(B) 2 and 4

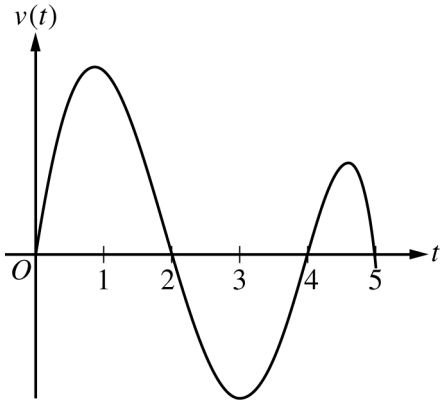
(C) 0 and 2 only

(D) 0, 1, and 2

GO ON TO THE NEXT PAGE.

78. The second derivative of a function f is given by $f''(x) = \sin(3x) - \cos(x^2)$. How many points of inflection does the graph of f have on the interval $0 < x < 3$?

(A) One (B) Three (C) Four (D) Five



79. Over the time interval $0 \le t \le 5$, a particle moves along the x-axis. The graph of the particle's velocity, v, is shown above. Over the time interval $0 \le t \le 5$, the particle's displacement is 3 and the particle travels a total distance of 13. What is the value of $\int_2^4 v(t)\,dt$?

(A) -10 (B) -5 (C) 5 (D) 10

GO ON TO THE NEXT PAGE.

80. The temperature in a room at midnight is 20 degrees Celsius. Over the next 24 hours, the temperature changes at a rate modeled by the differentiable function H, where $H(t)$ is measured in degrees Celsius per hour and time t is measured in hours since midnight. Which of the following is the best interpretation of $\int_0^6 H(t)\,dt$?

(A) The temperature of the room, in degrees Celsius, at 6:00 A.M.

(B) The average temperature of the room, in degrees Celsius, between midnight and 6:00 A.M.

(C) The change in the temperature of the room, in degrees Celsius, between midnight and 6:00 A.M.

(D) The rate at which the temperature in the room is changing, in degrees Celsius per hour, at 6:00 A.M.

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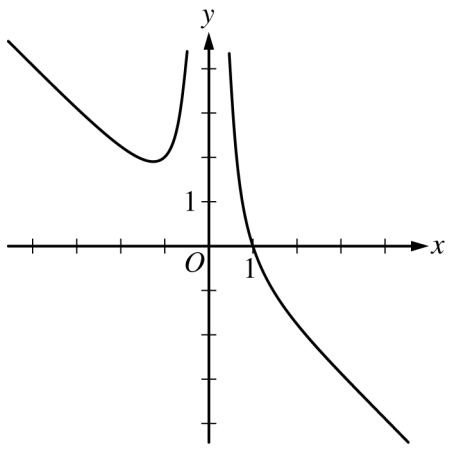
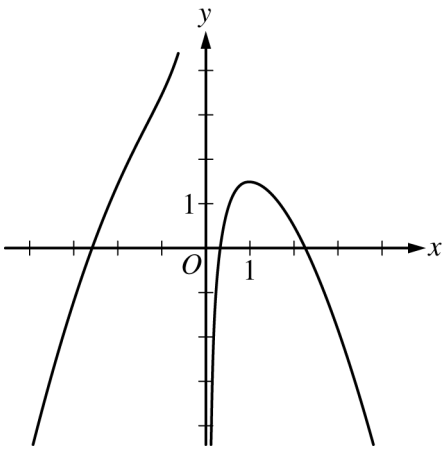


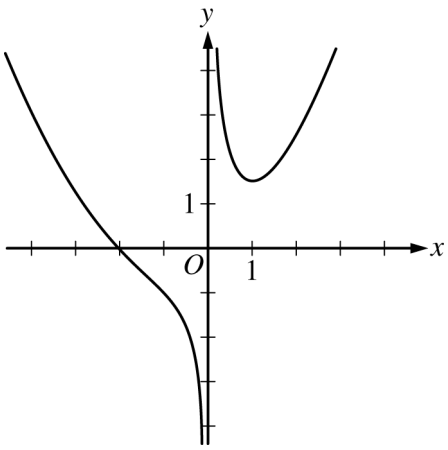
Graph of f'

81. The graph of f', the derivative of the function f, is shown above. Which of the following could be the graph of f ?

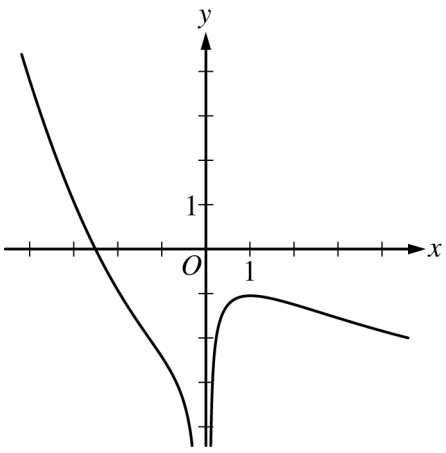
(A)



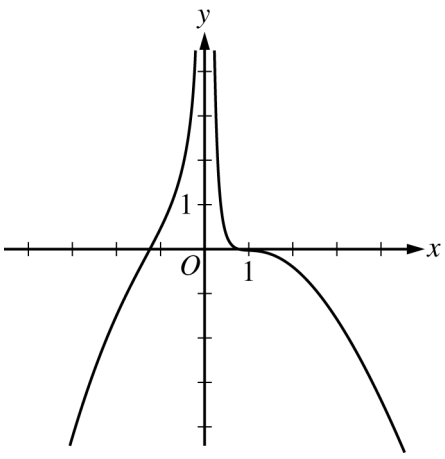
(B)



(C)



(D)



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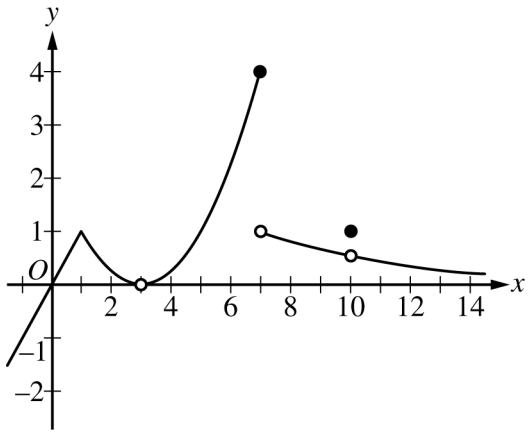
82. Let f be the function with derivative given by $f'(x) = \sin(x^2 - 3)$. At what values of x in the interval $-3 < x < 3$ does f have a relative maximum?

(A) -1.732 and 2.478 only

(B) -2.478 and 1.732 only

(C) -2.138, 0, and 2.138

(D) -2.478, -1.732, 1.732, and 2.478



Graph of f

83. The graph of the function f is shown above. At what value of x does f have a jump discontinuity?

(A) 1 (B) 3 (C) 7 (D) 10

GO ON TO THE NEXT PAGE.

84. Let f be a differentiable function such that $f(1) = \pi$ and $f'(x) = \sqrt{x^3 + 6}$. What is the value of $f(5)$?

(A) 11.941 (B) 14.587 (C) 24.672 (D) 27.814

---

85. People are entering a building at a rate modeled by $f(t)$ people per hour and exiting the building at a rate modeled by $g(t)$ people per hour, where t is measured in hours. The functions f and g are nonnegative and differentiable for all times t. Which of the following inequalities indicates that the rate of change of the number of people in the building is increasing at time t ?

(A) $f(t) > 0$

(B) $f'(t) > 0$

(C) $f(t) - g(t) > 0$

(D) $f'(t) - g'(t) > 0$

GO ON TO THE NEXT PAGE.

86. The velocity of a particle moving along the x-axis is given by $v(t) = \sqrt{t} - \cos(e^t)$ for $t \geq 0$. Which of the following statements describes the motion of the particle at $t = 1$?

(A) The particle is moving to the left with positive acceleration.

(B) The particle is moving to the right with positive acceleration.

(C) The particle is moving to the left with negative acceleration.

(D) The particle is moving to the right with negative acceleration.

---

87. A tire that is leaking air has an initial air pressure of 30 pounds per square inch (psi). The function $t = f(p)$ models the amount of time t, in hours, it takes for the air pressure of the tire to reach p psi. What are the units for $f'(p)$?

(A) hours (B) psi (C) psi per hour (D) hours per psi

88. The first derivative of the function f is defined by $f'(x) = \dfrac{x + 2e^{-x}}{x^2 + 0.7}$. On what intervals is f increasing?

(A) $-1.384 < x < -0.264$ only

(B) $x < -0.633$ and $x > 0.319$ only

(C) $-\infty < x < \infty$

(D) There are no intervals on which f is increasing.

x	0	4	6	8	13
$f(x)$	3	4.5	3	2.5	4.4

89. The table above shows selected values of a continuous function f. For $0 \le x \le 13$, what is the fewest possible number of times $f(x) = 4$?

(A) One (B) Two (C) Three (D) Four

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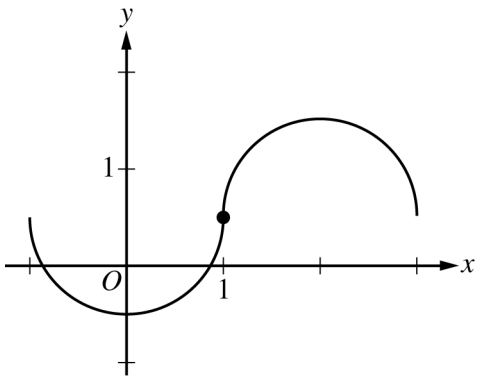

Graph of h'

90. The function h is defined on the closed interval $[-1, 3]$. The graph of h', the derivative of h, is shown above. The graph consists of two semicircles with a common endpoint at $x = 1$. Which of the following statements about h must be true?

I. $h(-1) = h(3)$

II. h is continuous at $x = 1$.

III. The graph of h has a vertical asymptote at $x = 1$.

(A) None (B) II only (C) I and II only (D) I and III only

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

AP® Calculus AB Exam

SECTION II: Free Response

DO NOT OPEN THIS BOOKLET OR BEGIN PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.3%

Part B

Number of Questions
4
Time
60 minutes
Electronic Device
None allowed
Percent of Section II Score
66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
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Instructions

The questions for Section II are printed in this booklet. Do not begin Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE PROBLEMS.

GO ON TO THE NEXT PAGE.

t (minutes)	0	2	5	7	10
$h(t)$ (inches)	3.5	10.0	15.5	18.5	20.0

1. The depth of water in tank A, in inches, is modeled by a differentiable and increasing function h for $0 \le t \le 10$, where t is measured in minutes. Values of $h(t)$ for selected values of t are given in the table above.

   (a) Use the data in the table to find an approximation for $h'(6)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Approximate the value of $\int_0^{10} h(t)\,dt$ using a right Riemann sum with the four subintervals indicated by the data in the table. Is this approximation greater than or less than $\int_0^{10} h(t)\,dt$? Give a reason for your answer.

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(c) The depth of water in tank B, in inches, is modeled by the function $g(t) = 3.2 + 17.5\sqrt{\sin(0.16t)}$ for $0 \le t \le 10$, where t is measured in minutes. Find the average depth of the water in tank B over the interval $0 \le t \le 10$. Is this value greater than or less than the average depth of the water in tank A over the interval $0 \le t \le 10$? Give a reason for your answer.

(d) According to the model given in part (c), is the depth of the water in tank B increasing or decreasing at time $t = 6$? Give a reason for your answer.

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2. Particle Q moves along the x-axis so that its velocity at any time t is given by $v_Q(t) = 1 - 3\cos\left(\frac{t^2}{5}\right)$, and its acceleration at any time t is given by $a_Q(t) = \frac{6t}{5}\sin\left(\frac{t^2}{5}\right)$. The particle is at position $x = 2$ at time $t = 0$.

(a) In the interval $0 < t < 5$, when is the velocity of particle Q increasing? Give a reason for your answer.

(b) Find the position of particle Q at time $t = 3$.

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(c) A second particle, R, moves along the x-axis so that its position at any time t is given by a differentiable function $x_R(t)$, where $x_R(1) = 4$ and $x_R(3) = 8$. Explain why there must be a time t, for $1 < t < 3$, at which the velocity of particle R is 2.

(d) At time $t = 3$, the velocity of particle R described in part (c) is -2. Are particles Q and R moving toward each other or away from each other at time $t = 3$? Explain your reasoning.

Do not write beyond this border.

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END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.