77. True. Let $y = ax^3 + bx^2 + cx + d, a \neq 0$. Then $y'' = 6ax + 2b = 0$ when $x = -(b/3a)$, and the concavity changes at this point.

78. False. $f(x) = 1/x$ has a discontinuity at $x = 0$.

79. False

$$f(x) = 3 \sin x + 2 \cos x$$
$$f'(x) = 3 \cos x - 2 \sin x$$
$$3 \cos x - 2 \sin x = 0$$
$$3 \cos x = 2 \sin x$$
$$\tfrac{3}{2} = \tan x$$

Critical number: $x = \tan^{-1}\left(\tfrac{3}{2}\right)$

$f\left(\tan^{-1} \tfrac{3}{2}\right) \approx 3.60555$ is the maximum value of y.

80. True

$$y = \sin(bx)$$

Slope: $y' = b \cos(bx)$

$-b \leq y' \leq b$ (Assume $b > 0$)

Section 3.5 Limits at Infinity

1. $f(x) = \dfrac{3x^2}{x^2 + 2}$

No vertical asymptotes

Horizontal asymptote: $y = 3$

Matches (f)

2. $f(x) = \dfrac{2x}{\sqrt{x^2 + 2}}$

No vertical asymptotes

Horizontal asymptotes: $y = \pm 2$

Matches (c)

3. $f(x) = \dfrac{x}{x^2 + 2}$

No vertical asymptotes

Horizontal asymptote: $y = 0$

Matches (d)

4. $f(x) = 2 + \dfrac{x^2}{x^4 + 1}$

No vertical asymptotes

Horizontal asymptote: $y = 2$

Matches (a)

5. $f(x) = \dfrac{4\sin x}{x^2 + 1}$

No vertical asymptotes

Horizontal asymptotes: $y = 0$

Matches (b)

6. $f(x) = \dfrac{2x^2 - 3x + 5}{x^2 + 1}$

No vertical asymptotes

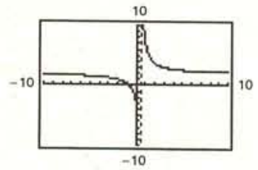
Horizontal asymptote: $y = 2$

Matches (e)

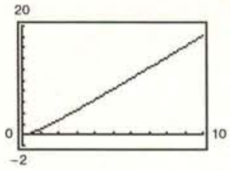
7. $f(x) = \dfrac{4x + 3}{2x - 1}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	7	2.26	2.025	2.0025	2.0003	2	2

$\lim_{x \to \infty} f(x) = 2$



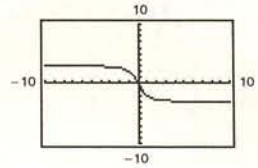
8. $f(x) = \dfrac{2x^2}{x+1}$



x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	1	18.18	198.02	1998.02	19,998	199,998	1,999,998

$\lim_{x\to\infty} f(x) = \infty$ (Limit does not exist.)

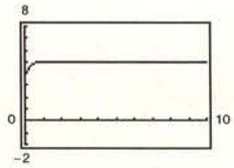
9. $f(x) = \dfrac{-6x}{\sqrt{4x^2+5}}$



x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	-2	-2.98	-2.9998	-3	-3	-3	-3

$\lim_{x\to\infty} f(x) = -3$

10. $f(x) = 5 - \dfrac{1}{x^2+1}$



x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	4.5	4.99	4.9999	4.999999	5	5	5

$\lim_{x\to\infty} f(x) = 5$

11. $\lim_{x\to\infty} \dfrac{2x-1}{3x+2} = \lim_{x\to\infty} \dfrac{2-(1/x)}{3+(2/x)} = \dfrac{2-0}{3+0} = \dfrac{2}{3}$

12. $\lim_{x\to\infty} \dfrac{5x^3+1}{10x^3-3x^2+7} = \lim_{x\to\infty} \dfrac{5+(1/x^3)}{10-(3/x)+(7/x^3)} = \dfrac{1}{2}$

13. $\lim_{x\to\infty} \dfrac{x}{x^2-1} = \lim_{x\to\infty} \dfrac{1/x}{1-(1/x^2)} = \dfrac{0}{1} = 0$

14. $\lim_{x\to\infty} \dfrac{2x^{10}-1}{10x^{11}-3} = \lim_{x\to\infty} \dfrac{(2/x)-(1/x^{11})}{10-(3/x^{11})} = \dfrac{0}{10} = 0$

15. $\lim_{x\to-\infty} \dfrac{5x^2}{x+3} = \lim_{x\to-\infty} \dfrac{5x}{1+(3/x)} = -\infty$

Limit does not exist.

16. $\lim_{x\to\infty} \left(2x - \dfrac{1}{x^2}\right) = \infty - 0 = \infty$

Limit does not exist.

17. $\lim_{x\to-\infty} \dfrac{x}{\sqrt{x^2-x}} = \lim_{x\to-\infty} \dfrac{1}{\dfrac{\sqrt{x^2-x}}{-\sqrt{x^2}}}$, $\left(\text{for } x < 0 \text{ we have } x = -\sqrt{x^2}\right)$

$= \lim_{x\to-\infty} \dfrac{-1}{\sqrt{1-(1/x)}} = -1$

18. $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \dfrac{1}{\sqrt{1+(1/x^2)}} = 1$

19. $\lim_{x\to\infty} \dfrac{2x+1}{\sqrt{x^2-x}} = \lim_{x\to\infty} \dfrac{2+(1/x)}{\sqrt{1-(1/x)}} = 2$

20. $\lim_{x\to-\infty} \frac{-3x+1}{\sqrt{x^2+x}} = \lim_{x\to-\infty} \frac{-3+(1/x)}{\frac{\sqrt{x^2+x}}{-\sqrt{x^2}}}$, (for $x < 0$ we have $-\sqrt{x^2} = x$)

$$= \lim_{x\to-\infty} \frac{3-(1/x)}{\sqrt{1+(1/x)}} = 3$$

21. Since $(-1/x) \le (\sin(2x))/x \le (1/x)$ for all $x \ne 0$, we have by the Squeeze Theorem,

$$\lim_{x\to\infty} -\frac{1}{x} \le \lim_{x\to\infty} \frac{\sin(2x)}{x} \le \lim_{x\to\infty} \frac{1}{x}$$

$$0 \le \lim_{x\to\infty} \frac{\sin(2x)}{x} \le 0.$$

Therefore, $\lim_{x\to\infty} \frac{\sin(2x)}{x} = 0$.

22. $\lim_{x\to\infty} \frac{x-\cos x}{x} = \lim_{x\to\infty} \left(1 - \frac{\cos x}{x}\right)$

$$= 1 - 0 = 1$$

Note:

$\lim_{x\to\infty} \frac{\cos x}{x} = 0$ by the Squeeze Theorem since

$$-\frac{1}{x} \le \frac{\cos x}{x} \le \frac{1}{x}.$$

23. $\lim_{x\to\infty} \frac{1}{2x+\sin x} = 0$

24. $\lim_{x\to\infty} \sin\frac{1}{x} = \sin 0 = 0$

25. $\lim_{x\to\infty} x\sin\frac{1}{x} = \lim_{t\to0^+} \frac{\sin t}{t} = 1$

(Let $x = 1/t$.)

26. $\lim_{x\to\infty} x\tan\frac{1}{x} = \lim_{t\to0^+} \frac{\tan t}{t} = \lim_{t\to0^+} \left[\frac{\sin t}{t} \cdot \frac{1}{\cos t}\right]$

$$= (1)(1) = 1$$

(Let $x = 1/t$.)

27. $\lim_{x\to-\infty} \left(x + \sqrt{x^2+3}\right) = \lim_{x\to-\infty} \left[\left(x+\sqrt{x^2+3}\right) \cdot \frac{x-\sqrt{x^2+3}}{x-\sqrt{x^2+3}}\right] = \lim_{x\to-\infty} \frac{-3}{x-\sqrt{x^2+3}} = 0$

28. $\lim_{x\to\infty} \left(2x - \sqrt{4x^2+1}\right) = \lim_{x\to\infty} \left[\left(2x-\sqrt{4x^2+1}\right) \cdot \frac{2x+\sqrt{4x^2+1}}{2x+\sqrt{4x^2+1}}\right] = \lim_{x\to\infty} \frac{-1}{2x+\sqrt{4x^2+1}} = 0$

29. $\lim_{x\to\infty} \left(x - \sqrt{x^2+x}\right) = \lim_{x\to\infty} \left[\left(x-\sqrt{x^2+x}\right) \cdot \frac{x+\sqrt{x^2+x}}{x+\sqrt{x^2+x}}\right]$

$$= \lim_{x\to\infty} \frac{-x}{x+\sqrt{x^2+x}} = \lim_{x\to\infty} \frac{-1}{1+\sqrt{1+(1/x)}} = -\frac{1}{2}$$

30. $\lim_{x\to-\infty} \left(3x + \sqrt{9x^2-x}\right) = \lim_{x\to-\infty} \left[\left(3x+\sqrt{9x^2-x}\right) \cdot \frac{3x-\sqrt{9x^2-x}}{3x-\sqrt{9x^2-x}}\right]$

$$= \lim_{x\to-\infty} \frac{x}{3x-\sqrt{9x^2-x}}$$

$$= \lim_{x\to-\infty} \frac{1}{3 - \frac{\sqrt{9x^2-x}}{-\sqrt{x^2}}}$$ (for $x < 0$ we have $x = -\sqrt{x^2}$)

$$= \lim_{x\to-\infty} \frac{1}{3+\sqrt{9-(1/x)}} = \frac{1}{6}$$