62. To show that $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is the midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) we must show that $d_1 = d_2$ and $d_1 + d_2 = d_3$ (see graph).

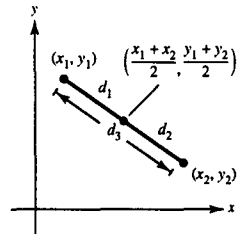
$$d_1 = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$

$$= \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2} = \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_2 = \sqrt{\left(x_2 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_2 - \frac{y_1 - y_2}{2}\right)^2}$$

$$= \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2} = \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_3 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Therefore, $d_1 = d_2$ and $d_1 + d_2 = d_3$.

Appendix A.3

1. (a) $396°, -324°$

(b) $240°, -480°$

2. (a) $660°, -60°$

(b) $300°, -60°$

3. (a) $\frac{19\pi}{9}, -\frac{17\pi}{9}$

(b) $\frac{10\pi}{3}, -\frac{2\pi}{3}$

4. (a) $\frac{7\pi}{4}, -\frac{\pi}{4}$

(b) $\frac{26\pi}{9}, -\frac{10\pi}{9}$

5. (a) $30\left(\frac{\pi}{180}\right) = \frac{\pi}{6} \approx 0.524$

(b) $150\left(\frac{\pi}{180}\right) = \frac{5\pi}{6} \approx 2.618$

(c) $315\left(\frac{\pi}{180}\right) = \frac{7\pi}{4} \approx 5.498$

(d) $120\left(\frac{\pi}{180}\right) = \frac{2\pi}{3} \approx 2.094$

6. (a) $-20\left(\frac{\pi}{180}\right) = -\frac{\pi}{9} \approx -0.349$

(b) $-240\left(\frac{\pi}{180}\right) = -\frac{4\pi}{3} \approx -4.189$

(c) $-270\left(\frac{\pi}{180}\right) = -\frac{3\pi}{2} \approx -4.712$

(d) $144\left(\frac{\pi}{180}\right) = \frac{4\pi}{5} \approx 2.513$

7. (a) $\frac{3\pi}{2}\left(\frac{180}{\pi}\right) = 270°$

(b) $\frac{7\pi}{6}\left(\frac{180}{\pi}\right) = 210°$

(c) $-\frac{7\pi}{12}\left(\frac{180}{\pi}\right) = -105°$

(d) $-2.637\left(\frac{180}{\pi}\right) \approx -135.6°$

8. (a) $\frac{7\pi}{3}\left(\frac{180}{\pi}\right) = 420°$

(b) $-\frac{11}{30}\left(\frac{180}{\pi}\right) = -66°$

(c) $\frac{11\pi}{6}\left(\frac{180}{\pi}\right) = 330°$

(d) $0.438\left(\frac{180}{\pi}\right) \approx 25.1°$

9.

r	8 ft	15 in.	85 cm	24 in.	$\frac{12963}{\pi}$ mi.
s	12 ft.	24 in.	63.72π	96 in.	8642 mi.
θ	1.5	1.6	$\frac{3\pi}{4}$	4	$\frac{2\pi}{3}$

10. (a) $50 \text{ mph} = \frac{50(5280)}{60} = 4400 \text{ ft/min}$

Circumference of tire: $C = 2.5\pi$ feet

Revolutions per minute: $\frac{4400}{2.5\pi} \approx 560.2$

(b) $\theta = \frac{4400}{2.5\pi}(2\pi) = 3520$ radians

Angular speed: $\frac{\theta}{t} = \frac{3520 \text{ radians}}{1 \text{ minute}} = 3520 \text{ rad/min}$

11. (a) $x = 3, y = 4, r = 5$

$\sin\theta = \frac{4}{5}$ $\quad \csc\theta = \frac{5}{4}$

$\cos\theta = \frac{3}{5}$ $\quad \sec\theta = \frac{5}{3}$

$\tan\theta = \frac{4}{3}$ $\quad \cot\theta = \frac{3}{4}$

(b) $x = -12, y = -5, r = 13$

$\sin\theta = -\frac{5}{13}$ $\quad \csc\theta = -\frac{13}{5}$

$\cos\theta = -\frac{12}{13}$ $\quad \sec\theta = -\frac{13}{12}$

$\tan\theta = \frac{5}{12}$ $\quad \cot\theta = \frac{12}{5}$

12. (a) $x = 8, y = -15, r = 17$

$\sin\theta = -\frac{15}{17}$ $\quad \csc\theta = -\frac{17}{15}$

$\cos\theta = \frac{8}{17}$ $\quad \sec\theta = \frac{17}{8}$

$\tan\theta = -\frac{15}{8}$ $\quad \cot\theta = -\frac{8}{15}$

(b) $x = 1, y = -1, r = \sqrt{2}$

$\sin\theta = -\frac{\sqrt{2}}{2}$ $\quad \csc\theta = -\sqrt{2}$

$0\cos\theta = \frac{\sqrt{2}}{2}$ $\quad \sec\theta = \sqrt{2}$

$\tan\theta = -1$ $\quad \cot\theta = -1$

13. (a) $\sin\theta < 0 \Rightarrow \theta$ is in Quadrant III or IV.

$\cos\theta < 0 \Rightarrow \theta$ is in Quadrant II or III.

$\sin\theta < 0$ **and** $\cos\theta < 0 \Rightarrow \theta$ is in Quadrant III.

(b) $\sec\theta > 0 \Rightarrow \theta$ is in Quadrant I or IV.

$\cot\theta < 0 \Rightarrow \theta$ is in Quadrant II or IV.

$\sec\theta > 0$ **and** $\cot\theta < 0 \Rightarrow \theta$ is in Quadrant IV.

14. (a) $\sin\theta > 0 \Rightarrow \theta$ is in Quadrant I or II.

$\cos\theta < 0 \Rightarrow \theta$ is in Quadrant II or III.

$\sin\theta > 0$ **and** $\operatorname{ocs}\theta < 0 \Rightarrow \theta$ is in Quadrant II.

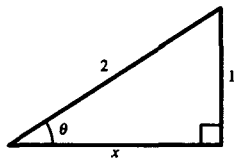
(b) $\csc\theta < 0 \Rightarrow \theta$ is in Quadrant III or IV.

$\tan\theta > 0 \Rightarrow \theta$ is in Quadrant I or III.

$\csc\theta < 0$ **and** $\tan\theta > 0 \Rightarrow \theta$ is in Quadrant III.

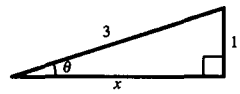
15. $x^2 + 1^2 = 2^2 \Rightarrow x = \sqrt{3}$

$\cos\theta = \frac{x}{2} = \frac{\sqrt{3}}{2}$



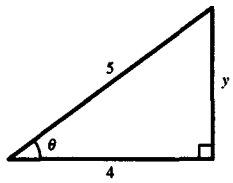
16. $x^2 + 1^2 - 3^2 \Rightarrow x = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \frac{1}{x} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$



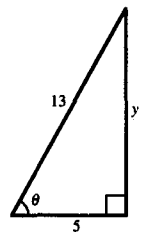
17. $4^2 + y^2 = 5^2 \Rightarrow y = 3$

$\cot\theta = \frac{4}{y} = \frac{4}{3}$



18. $5^2 + y^2 = 13^2 \Rightarrow y = 12$

$\csc\theta = \frac{13}{y} = \frac{13}{12}$



19. (a) $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \sqrt{3}$

(b) $\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos 120° = -\cos 60° = -\frac{1}{2}$

$\tan 120° = -\tan 60° = -\sqrt{3}$

(c) $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\tan\frac{\pi}{4} = 1$

(d) $\sin\frac{5\pi}{4} - \sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\cos\frac{5\pi}{4} = \cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\tan\frac{5\pi}{4} = \tan\frac{\pi}{4} = 1$

20. (a) $\sin(-30°) = -\sin 30° = \frac{1}{2}$

$\cos(-30°) = \cos 30° = \frac{\sqrt{3}}{2}$

$\tan(-30°) = -\tan 30° = -\frac{\sqrt{3}}{3}$

(b) $\sin 150° = \sin 30° = \frac{1}{2}$

$\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\tan 150° = -\tan 30° = -\frac{\sqrt{3}}{3}$

(c) $\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$

$\cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\tan\left(-\frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$

(d) $\sin\frac{\pi}{2} = 1$

$\cos\frac{\pi}{2} = 0$

$\tan\frac{\pi}{2}$ is undefined.

21. (a) $\sin 225° = -\sin 45° = -\frac{\sqrt{2}}{2}$

$\cos 225° = -\cos 45° = -\frac{\sqrt{2}}{2}$

$\tan 225° = \tan 45° = 1$

(b) $\sin(-225°) = \sin 45° = \frac{\sqrt{2}}{2}$

$\cos(-225°) = -\cos 45° = \frac{\sqrt{2}}{2}$

$\tan(-225°) = -\tan 45 = -1$

(c) $\sin\frac{5\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

$\cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$

$\tan\frac{5\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$

(d) $\sin\frac{11\pi}{6} = -\sin\frac{\pi}{6} = -\frac{1}{2}$

$\cos\frac{11\pi}{6} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\tan\frac{11\pi}{6} = -\tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$

22. (a) $\sin 750° = \sin 30° = \frac{1}{2}$

$\cos 750° = \cos 30° = \frac{\sqrt{3}}{2}$

$\tan 750° = \tan 30° = \frac{\sqrt{3}}{3}$

(b) $\sin 510° = \sin 30° = \frac{1}{2}$

$\cos 510° = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\tan 510° = -\tan 30° = -\frac{\sqrt{3}}{3}$

(c) $\sin\frac{10\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

$\cos\frac{10\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$

$\tan\frac{10\pi}{3} = \tan\frac{\pi}{3} = \sqrt{3}$

(d) $\sin\frac{17\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

$\cos\frac{17\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$

atan $\frac{17\pi}{3} = -\tan\frac{\pi}{3} = \sqrt{3}$

23. (a) $\sin 10° \approx 0.1736$

(b) $\csc 10° \approx 5.759$

24. (a) $\sec 225° \approx -1.414$

(b) $\sec 135° \approx -1.414$

25. (a) $\tan\frac{\pi}{9} \approx 0.3640$

(b) $\tan\frac{10\pi}{9} \approx 0.3640$

26. (a) $\cot 1.35 \approx 0.2245$

(b) $\tan 1.35 \approx 4.455$

27. (a) $\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

(b) $\cos\theta = -\frac{\sqrt{2}}{2}$

$\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$

28. (a) $\sec\theta = 2$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

(b) $\sec\theta = -2$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

29. (a) $\tan\theta = 1$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

(b) $\cot\theta = -\sqrt{3}$

$$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$$

30. (a) $\sin\theta = \frac{\sqrt{3}}{2}$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

(b) $\sin\theta = -\frac{\sqrt{3}}{2}$

$$\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$$

31. $2\sin^2\theta = 1$

$$\sin\theta = \pm\frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

32. $\tan^2\theta = 3$

$$\tan\theta = \pm\sqrt{3}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

33. $\tan^2\theta = \tan\theta = 0$

$$\tan\theta(\tan\theta - 1) = 0$$

$$\tan\theta = 0 \qquad \tan\theta = 1$$

$$\theta = 0, \pi \qquad \theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

34. $2\cos^2\theta - \cos\theta - 1 = 0$

$$(2\cos\theta + 1)(\cos\theta - 1) = 0$$

$$\cos\theta = -\frac{1}{2} \qquad \cos\theta = 1$$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3} \qquad \theta = 0$$

35. $\sec\theta\csc\theta - 2\csc\theta = 0$

$$\csc\theta(\sec\theta - 2) = 0$$

($\csc\theta \neq 0$ for any value of θ)

$$\sec\theta = 2$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

36. $\sin\theta = \cos\theta$

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

37. $\cos^2\theta + \sin\theta = 1$

$$1 - \sin^2\theta + \sin\theta = 1$$

$$\sin^2\theta - \sin\theta = 0$$

$$\sin\theta(\sin\theta - 1) = 0$$

$$\sin\theta = 0 \qquad \sin\theta = 1$$

$$\theta = 0, \pi \qquad \theta = \frac{\pi}{2}$$

38. $\cos\left(\frac{\pi}{2}\right) - \cos\theta = 1$

$$\cos\left(\frac{\theta}{2}\right) = \cos\theta + 1$$

$$\sqrt{\left(\frac{1}{2}\right)(1 + \cos\theta)} = \cos\theta + 1$$

$$\left(\frac{1}{2}\right)(1 + \cos\theta) = \cos^2\theta + 2\cos\theta + 1$$

$$0 = \cos^2\theta + \left(\frac{3}{2}\right)\cos\theta + \left(\frac{1}{2}\right)$$

$$0 = \left(\frac{1}{2}\right)(2\cos^2\theta + 3\cos\theta + 1)$$

$$0 = \left(\frac{1}{2}\right)(2\cos\theta + 1)(\cos\theta + 1)$$

$$\cos\theta = -\frac{1}{2} \qquad \cos\theta = -1$$

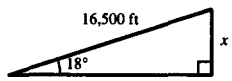
$$\theta = \frac{2\pi}{3} \qquad \theta = \pi$$

($\theta = 4\pi/3$ is extraneous)

39. (275 ft/sec)(60 sec) = 16,500 feet

$$\sin 18° = \frac{a}{16{,}500}$$

$$a = 16{,}500 \sin 18° \approx 5099 \text{ feet}$$



50. —CONTINUED—

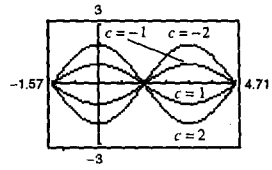
(c) $f(x) = c\cos x$; changing c changes the amplitude.

When $c = -2$: $f(x) = -2\cos x$.

When $c = -1$: $f(x) = -\cos x$.

When $c = 1$: $f(x) = \cos x$.

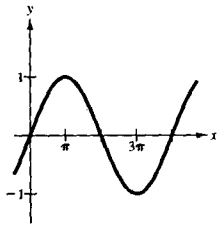
When $c = 2$: $f(x) = 2\cos x$.



51. $y = \sin\frac{x}{2}$

Period: 4π

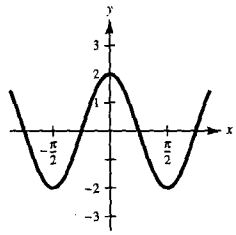
Amplitude: 1



52. $y = 2\cos 2x$

Period: π

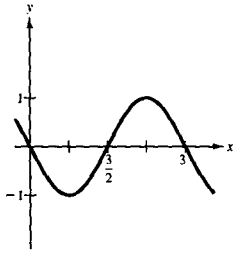
Amplitude: 2



53. $y = -\sin\frac{2\pi x}{3}$

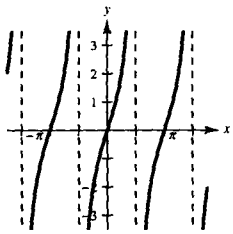
Period: 3

Amplitude: 1



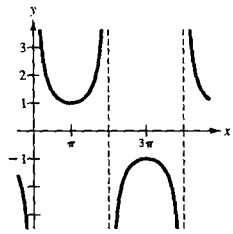
54. $y = 2\tan x$

Period: π



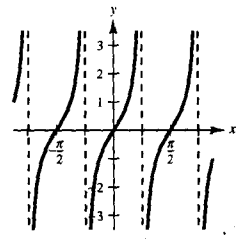
55. $y = \csc\frac{x}{2}$

Period: 4π



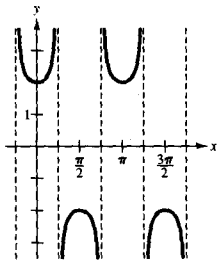
56. $y = \tan 2x$

Period: $\frac{\pi}{2}$



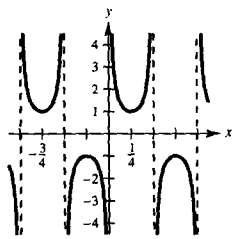
57. $y = 2\sec 2x$

Period: π



58. $y = \csc 2\pi x$

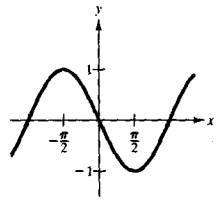
Period: 1



59. $y = \sin(x + \pi)$

Period: 2π

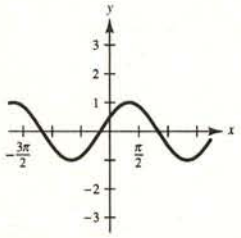
Amplitude: 1



60. $y = \cos\left(x - \frac{\pi}{3}\right)$

Period: 2π

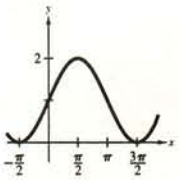
Amplitude: 1



61. $y = 1 + \cos\left(x - \frac{\pi}{2}\right)$

Period: 2π

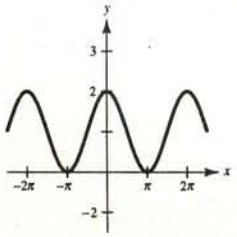
Amplitude: 1



62. $y = 1 + \sin\left(x + \frac{\pi}{2}\right)$

Period: 2π

Amplitude: 1



63. $y = a\cos(bx - c)$

From the graph, we see that the amplitude is 3, the period is 4π and the horizontal shift is, π. Thus,

$$a = 3$$

$$\frac{2\pi}{b} = 4\pi \Longrightarrow b = \frac{1}{2}$$

$$\frac{c}{d} = \pi \Longrightarrow c = \frac{\pi}{2}.$$

Therefore, $y = 3\cos[(1/2)x - (\pi/2)]$.

64. $y = a\sin(bx - c)$

From the graph, we see that the amplitude is $\frac{1}{2}$, the period is π and the horizontal shift is 0. Also, the graph is reflected about the x-axis. Thus,

$$a = -\frac{1}{2}$$

$$\frac{2\pi}{b} = \pi \Longrightarrow b = 2$$

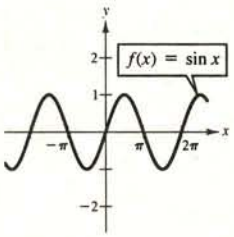
$$\frac{c}{b} = 0 \Longrightarrow c = 0.$$

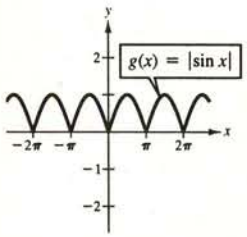
Therefore, $y = -\frac{1}{2}\sin 2x$.

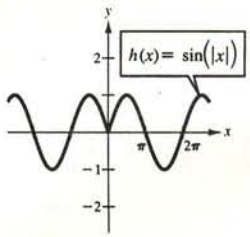
65. $f(x) = \sin x$

$g(x) = |\sin x|$

$h(x) = \sin|x|$





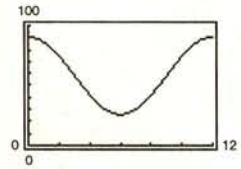


The graph of $|f(x)|$ will reflect any parts of the graph below the x-axis about the y-axis.

The graph of $f(|x|)$ will reflect the part of the graph to the left of the y-axis about the x-axis.

66. If $h = 51 + 50\sin\left(8\pi t - \frac{\pi}{2}\right)$, then $h = 1$ when $t = 0$.

67. $S = 58.3 + 32.5\cos\frac{\pi t}{6}$



Sales exceed 75,000 during the months of January, November, and December.