Antiderivatives and Indefinite Integration

w-up: AP multiple choice # 85(calculator)

If $f'(x)=x^2$ then what could $f(x)$ be?

Antiderivative: a function F is called an antiderivative of the function f if for every x in the domain of f, $F'(x)=f(x)$

So, the antiderivative of the function $f(x)=x^2$ is ....$F(x)=\frac{1}{3}x^3$ or $F(x)=\frac{1}{3}x^3+5$

or $F(x)=\frac{1}{3}x^3-9$ or ..... in general $F(x)=\frac{1}{3}x^3+C$

The process of finding the antiderivative is called INTEGRATION and is denoted by the symbol $\int$ called "the integral symbol"

$$\int f(x)\,dx = F(x)+C$$

Idefinite integral = antiderivative +C

Note: the +C must be placed on all antiderivatives(indefinite integrals) because the derivative of any constant is ZERO.

Properties of Integrals

$$\int kf(x)\,dx = k\int f(x)\,dx \text{ where } k \text{ is any constant}$$

$$\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$$

Basic Rules of Integration

$$\int 0\,dx = C$$

$$\int K\,dx = Kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \text{ for singular powers of } x$$

NOTE: there are NO other rules for integration such as the product rule, quotient rule and chain rule! Be sure to rewrite products and quotients as a singular power of x where possible. When this cannot be done, U-substitution will be attempted next(to be learned later this chapter).

EX) $\int 5x^3 - 3x^2\,dx$

EX) $\int \frac{1}{p^4}\,dp$

EX) $\int \sqrt{s} + 5\,ds$

EX) $\int \frac{x+1}{\sqrt{x}}\,dx$

Since we have learned the derivatives of all of the trigonometric functions we will also know six key integrals(reversals of these). Note: the negative can always be moved out in front of the integral due to the scalar property.

Integrals of Trigonometric Functions

$$\int \sin x\, dx = -\cos x + C \qquad \int \csc^2 x\, dx = -\cot x + C$$

$$\int \cos x\, dx = \sin x + C \qquad \int \sec x \tan x\, dx = \sec x + C$$

$$\int \sec^2 x\, dx = \tan x + C \qquad \int \csc x \cot x\, dx = -\csc x + C$$

To evaluate integrals containing trigonometric functions, use identities, properties and algebra to create integrals from the above list.

EX) $\int \dfrac{\sin x}{1-\sin^2 x}\, dx$

EX) $\int -1 - \cot^2 x\, dx$