



## **AP<sup>®</sup> Calculus AB 2012 Scoring Guidelines**

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**Question 1**

$t$ (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time  $t$  is modeled by a strictly increasing, twice-differentiable function  $W$ , where  $W(t)$  is measured in degrees Fahrenheit and  $t$  is measured in minutes. At time  $t = 0$ , the temperature of the water is  $55^\circ\text{F}$ . The water is heated for 30 minutes, beginning at time  $t = 0$ . Values of  $W(t)$  at selected times  $t$  for the first 20 minutes are given in the table above.

- (a) Use the data in the table to estimate  $W'(12)$ . Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate  $\int_0^{20} W'(t) dt$ . Using correct units, interpret the meaning of  $\int_0^{20} W'(t) dt$  in the context of this problem.
- (c) For  $0 \leq t \leq 20$ , the average temperature of the water in the tub is  $\frac{1}{20} \int_0^{20} W(t) dt$ . Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate  $\frac{1}{20} \int_0^{20} W(t) dt$ . Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For  $20 \leq t \leq 25$ , the function  $W$  that models the water temperature has first derivative given by  $W'(t) = 0.4\sqrt{t} \cos(0.06t)$ . Based on the model, what is the temperature of the water at time  $t = 25$ ?

(a) 
$$W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = \frac{67.9 - 61.8}{6}$$

$$= 1.017 \text{ (or 1.016)}$$

The water temperature is increasing at a rate of approximately  $1.017^\circ\text{F}$  per minute at time  $t = 12$  minutes.

(b) 
$$\int_0^{20} W'(t) dt = W(20) - W(0) = 71.0 - 55.0 = 16$$

The water has warmed by  $16^\circ\text{F}$  over the interval from  $t = 0$  to  $t = 20$  minutes.

(c) 
$$\frac{1}{20} \int_0^{20} W(t) dt \approx \frac{1}{20} (4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15))$$

$$= \frac{1}{20} (4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9)$$

$$= \frac{1}{20} \cdot 1215.8 = 60.79$$

This approximation is an underestimate, because a left Riemann sum is used and the function  $W$  is strictly increasing.

(d) 
$$W(25) = 71.0 + \int_{20}^{25} W'(t) dt$$

$$= 71.0 + 2.043155 = 73.043$$

2 :  $\begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

2 :  $\begin{cases} 1 : \text{value} \\ 1 : \text{interpretation with units} \end{cases}$

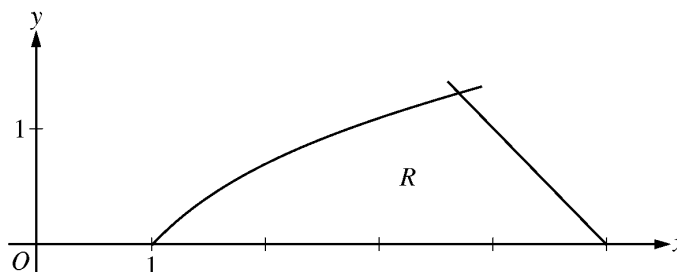
3 :  $\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{underestimate with reason} \end{cases}$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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**Question 2**

Let  $R$  be the region in the first quadrant bounded by the  $x$ -axis and the graphs of  $y = \ln x$  and  $y = 5 - x$ , as shown in the figure above.



- (a) Find the area of  $R$ .
- (b) Region  $R$  is the base of a solid. For the solid, each cross section perpendicular to the  $x$ -axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.
- (c) The horizontal line  $y = k$  divides  $R$  into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of  $k$ .

$$\ln x = 5 - x \Rightarrow x = 3.69344$$

Therefore, the graphs of  $y = \ln x$  and  $y = 5 - x$  intersect in the first quadrant at the point  $(A, B) = (3.69344, 1.30656)$ .

(a) 
$$\text{Area} = \int_0^B (5 - y - e^y) dy$$

$$= 2.986 \text{ (or } 2.985)$$

3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

OR

$$\text{Area} = \int_1^A \ln x \, dx + \int_A^5 (5 - x) \, dx$$

$$= 2.986 \text{ (or } 2.985)$$

(b) 
$$\text{Volume} = \int_1^A (\ln x)^2 \, dx + \int_A^5 (5 - x)^2 \, dx$$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{expression for total volume} \end{cases}$

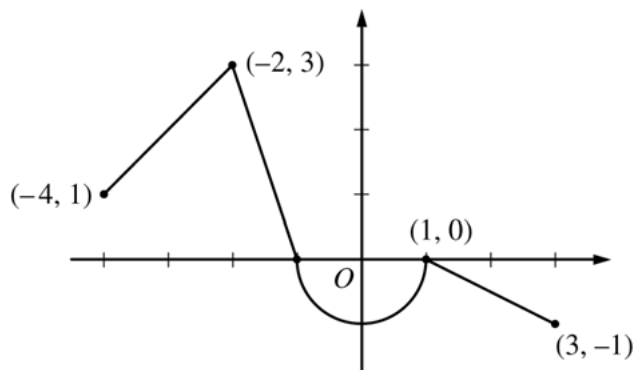
(c) 
$$\int_0^k (5 - y - e^y) \, dy = \frac{1}{2} \cdot 2.986 \text{ (or } \frac{1}{2} \cdot 2.985)$$

3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{equation} \end{cases}$

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**Question 3**

Let  $f$  be the continuous function defined on  $[-4, 3]$  whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let  $g$  be the function given by  $g(x) = \int_1^x f(t) dt$ .



Graph of  $f$

- (a) Find the values of  $g(2)$  and  $g(-2)$ .
- (b) For each of  $g'(-3)$  and  $g''(-3)$ , find the value or state that it does not exist.
- (c) Find the  $x$ -coordinate of each point at which the graph of  $g$  has a horizontal tangent line. For each of these points, determine whether  $g$  has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.
- (d) For  $-4 < x < 3$ , find all values of  $x$  for which the graph of  $g$  has a point of inflection. Explain your reasoning.

(a)  $g(2) = \int_1^2 f(t) dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$   
 $g(-2) = \int_1^{-2} f(t) dt = -\int_{-2}^1 f(t) dt$   
 $= -\left(\frac{3}{2} - \frac{\pi}{2}\right) = \frac{\pi}{2} - \frac{3}{2}$

2 :  $\begin{cases} 1 : g(2) \\ 1 : g(-2) \end{cases}$

(b)  $g'(x) = f(x) \Rightarrow g'(-3) = f(-3) = 2$   
 $g''(x) = f'(x) \Rightarrow g''(-3) = f'(-3) = 1$

2 :  $\begin{cases} 1 : g'(-3) \\ 1 : g''(-3) \end{cases}$

(c) The graph of  $g$  has a horizontal tangent line where  $g'(x) = f(x) = 0$ . This occurs at  $x = -1$  and  $x = 1$ .

$g'(x)$  changes sign from positive to negative at  $x = -1$ . Therefore,  $g$  has a relative maximum at  $x = -1$ .

$g'(x)$  does not change sign at  $x = 1$ . Therefore,  $g$  has neither a relative maximum nor a relative minimum at  $x = 1$ .

3 :  $\begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : x = -1 \text{ and } x = 1 \\ 1 : \text{answers with justifications} \end{cases}$

(d) The graph of  $g$  has a point of inflection at each of  $x = -2$ ,  $x = 0$ , and  $x = 1$  because  $g''(x) = f'(x)$  changes sign at each of these values.

2 :  $\begin{cases} 1 : \text{answer} \\ 1 : \text{explanation} \end{cases}$

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**Question 4**

The function  $f$  is defined by  $f(x) = \sqrt{25 - x^2}$  for  $-5 \leq x \leq 5$ .

- (a) Find  $f'(x)$ .
- (b) Write an equation for the line tangent to the graph of  $f$  at  $x = -3$ .
- (c) Let  $g$  be the function defined by  $g(x) = \begin{cases} f(x) & \text{for } -5 \leq x \leq -3 \\ x + 7 & \text{for } -3 < x \leq 5. \end{cases}$   
Is  $g$  continuous at  $x = -3$ ? Use the definition of continuity to explain your answer.
- (d) Find the value of  $\int_0^5 x\sqrt{25 - x^2} \, dx$ .

(a)  $f'(x) = \frac{1}{2}(25 - x^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}, \quad -5 < x < 5$

2 :  $f'(x)$

(b)  $f'(-3) = \frac{3}{\sqrt{25 - 9}} = \frac{3}{4}$

$f(-3) = \sqrt{25 - 9} = 4$

An equation for the tangent line is  $y = 4 + \frac{3}{4}(x + 3)$ .

2 :  $\begin{cases} 1 : f'(-3) \\ 1 : \text{answer} \end{cases}$

(c)  $\lim_{x \rightarrow -3^-} g(x) = \lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^-} \sqrt{25 - x^2} = 4$

$\lim_{x \rightarrow -3^+} g(x) = \lim_{x \rightarrow -3^+} (x + 7) = 4$

Therefore,  $\lim_{x \rightarrow -3} g(x) = 4$ .

$g(-3) = f(-3) = 4$

So,  $\lim_{x \rightarrow -3} g(x) = g(-3)$ .

Therefore,  $g$  is continuous at  $x = -3$ .

2 :  $\begin{cases} 1 : \text{considers one-sided limits} \\ 1 : \text{answer with explanation} \end{cases}$

(d) Let  $u = 25 - x^2 \Rightarrow du = -2x \, dx$

$\int_0^5 x\sqrt{25 - x^2} \, dx = -\frac{1}{2} \int_{25}^0 \sqrt{u} \, du$

$= \left[ -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} \right]_{u=25}^{u=0}$

$= -\frac{1}{3}(0 - 125) = \frac{125}{3}$

3 :  $\begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

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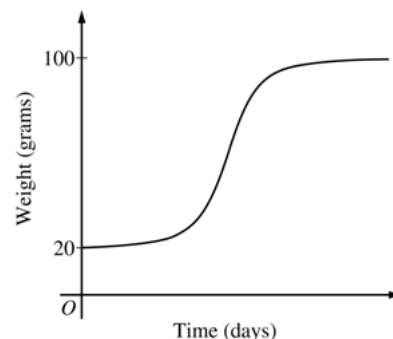
**Question 5**

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time  $t = 0$ , when the bird is first weighed, its weight is 20 grams. If  $B(t)$  is the weight of the bird, in grams, at time  $t$  days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let  $y = B(t)$  be the solution to the differential equation above with initial condition  $B(0) = 20$ .

- (a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.
- (b) Find  $\frac{d^2B}{dt^2}$  in terms of  $B$ . Use  $\frac{d^2B}{dt^2}$  to explain why the graph of  $B$  cannot resemble the following graph.
- (c) Use separation of variables to find  $y = B(t)$ , the particular solution to the differential equation with initial condition  $B(0) = 20$ .



(a)  $\left. \frac{dB}{dt} \right|_{B=40} = \frac{1}{5}(60) = 12$

$$\left. \frac{dB}{dt} \right|_{B=70} = \frac{1}{5}(30) = 6$$

Because  $\left. \frac{dB}{dt} \right|_{B=40} > \left. \frac{dB}{dt} \right|_{B=70}$ , the bird is gaining weight faster when it weighs 40 grams.

(b)  $\frac{d^2B}{dt^2} = -\frac{1}{5} \frac{dB}{dt} = -\frac{1}{5} \cdot \frac{1}{5}(100 - B) = -\frac{1}{25}(100 - B)$

Therefore, the graph of  $B$  is concave down for  $20 \leq B < 100$ . A portion of the given graph is concave up.

(c)  $\frac{dB}{dt} = \frac{1}{5}(100 - B)$

$$\int \frac{1}{100 - B} dB = \int \frac{1}{5} dt$$

$$-\ln|100 - B| = \frac{1}{5}t + C$$

Because  $20 \leq B < 100$ ,  $|100 - B| = 100 - B$ .

$$-\ln(100 - 20) = \frac{1}{5}(0) + C \Rightarrow -\ln(80) = C$$

$$100 - B = 80e^{-t/5}$$

$$B(t) = 100 - 80e^{-t/5}, \quad t \geq 0$$

2 :  $\left\{ \begin{array}{l} 1 : \text{uses } \frac{dB}{dt} \\ 1 : \text{answer with reason} \end{array} \right.$

2 :  $\left\{ \begin{array}{l} 1 : \frac{d^2B}{dt^2} \text{ in terms of } B \\ 1 : \text{explanation} \end{array} \right.$

5 :  $\left\{ \begin{array}{l} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } B \end{array} \right.$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

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**Question 6**

For  $0 \leq t \leq 12$ , a particle moves along the  $x$ -axis. The velocity of the particle at time  $t$  is given by  $v(t) = \cos\left(\frac{\pi}{6}t\right)$ . The particle is at position  $x = -2$  at time  $t = 0$ .

- (a) For  $0 \leq t \leq 12$ , when is the particle moving to the left?
- (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time  $t = 0$  to time  $t = 6$ .
- (c) Find the acceleration of the particle at time  $t$ . Is the speed of the particle increasing, decreasing, or neither at time  $t = 4$ ? Explain your reasoning.
- (d) Find the position of the particle at time  $t = 4$ .

(a)  $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \Rightarrow t = 3, 9$

The particle is moving to the left when  $v(t) < 0$ .  
This occurs when  $3 < t < 9$ .

2 :  $\begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{interval} \end{cases}$

(b)  $\int_0^6 |v(t)| dt$

1 : answer

(c)  $a(t) = -\frac{\pi}{6} \sin\left(\frac{\pi}{6}t\right)$

$$a(4) = -\frac{\pi}{6} \sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$$

$$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$$

The speed is increasing at time  $t = 4$ , because velocity and acceleration have the same sign.

3 :  $\begin{cases} 1 : a(t) \\ 2 : \text{conclusion with reason} \end{cases}$

(d)  $x(4) = -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right) dt$

$$= -2 + \left[ \frac{6}{\pi} \sin\left(\frac{\pi}{6}t\right) \right]_0^4$$

$$= -2 + \frac{6}{\pi} \left[ \sin\left(\frac{2\pi}{3}\right) - 0 \right]$$

$$= -2 + \frac{6}{\pi} \cdot \frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi}$$

3 :  $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$