Arc Length and Surfaces of Revolution

W-up: Find the distance between (-4, -7) and (1,8)

Arc Length of a smooth curve on the interval [a, b]

$$S=\int_a^b \sqrt{1+[f'(x)]^2}\,dx \quad \text{for } y=f(x)$$

or

$$S=\int_c^d \sqrt{1+[g'(y)]^2}\,dy \quad \text{for } x=g(y)$$

EX) Find the arc length of the curve $y=x^2+1$ over [0, 3].

EX) Find the arc length of the curve $x=y^2-3,\ y\geq 0$ over [0, 6].

Surface Area of Solids of Revolution

$$S=2\pi\int_a^b r(x)\bullet\sqrt{1+[f'(x)]^2}\,dx$$

or

$$S=2\pi\int_c^d r(y)\bullet\sqrt{1+[g'(y)]^2}\,dy$$

Where $r(x)$ & $r(y)$ is the distance from f to the axis of rotation

Note: Either formula is true for both horizontal and vertical rotations!

EX) Find the area of the surface of revolution for $f(x)=x^3$ over [0, 1] rotated about the ***x*-axis**.

EX) Find the area of the surface of revolution for $f(x)=x^3$ over [0, 1] rotated about the ***y*-axis**.

EX) Find the area of the surface of revolution for $f(x)=x^3$ over [0, 1] rotated about the line $y = -1$.

EX) Find the area of the surface of revolution for $f(x)=e^x$ over [0, 2] rotated about: A) the x-axis B) the y-axis.