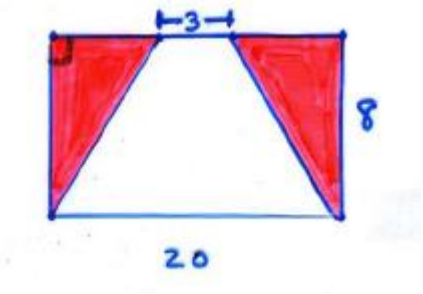
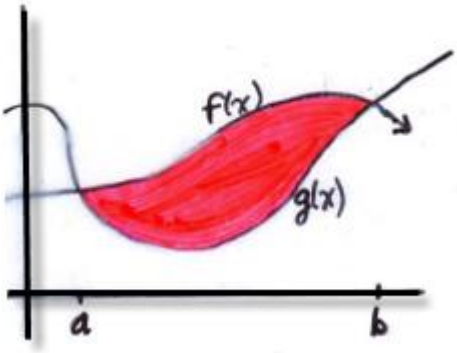
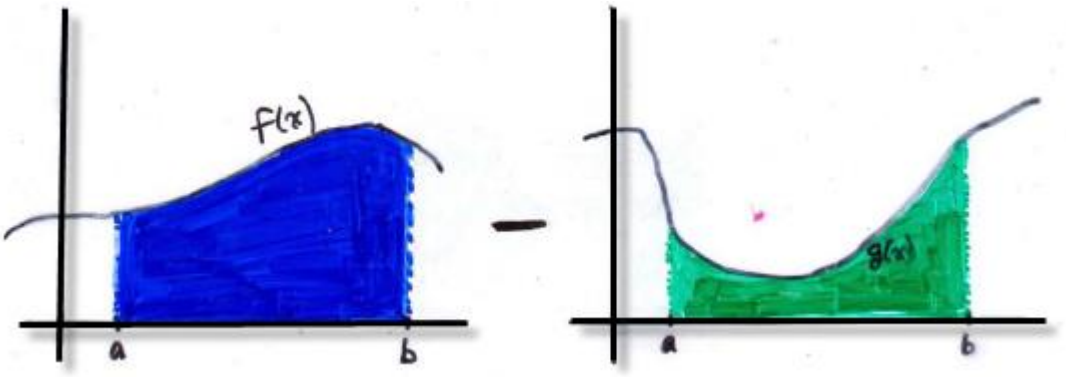
Area of a Region Between Curves

W-up: Find the area of the shaded region



Area of Irregular Regions





$$\int_a^b f(x)dx \quad - \quad \int_a^b g(x)dx$$

So, area between two functions can be found using:

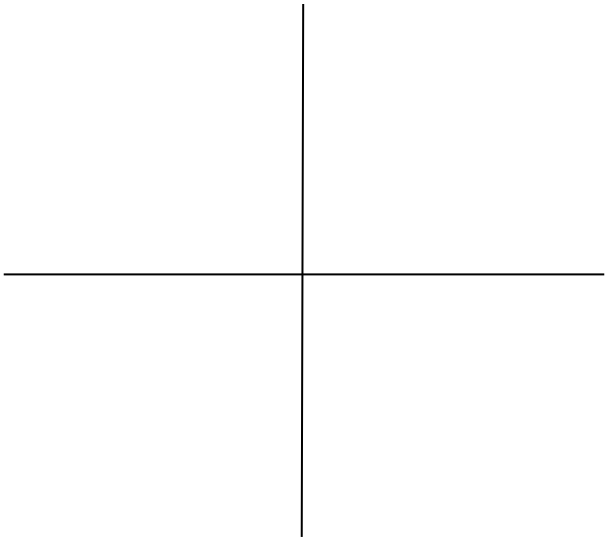
$A = \int_a^b \left[f(x) - g(x) \right] dx$ if f and g are continuous functions, $f(x) \geq g(x)$ and $\boldsymbol{a}$ & $\boldsymbol{b}$ represent the x-coordinates of the points of intersection of the curves.

Geometrically speaking, "TOP CURVE – BOTTOM CURVE"

Note: this works **anytime** integrals with respect to x are used regardless of whether the functions are above or below the x-axis. Click here to see why!

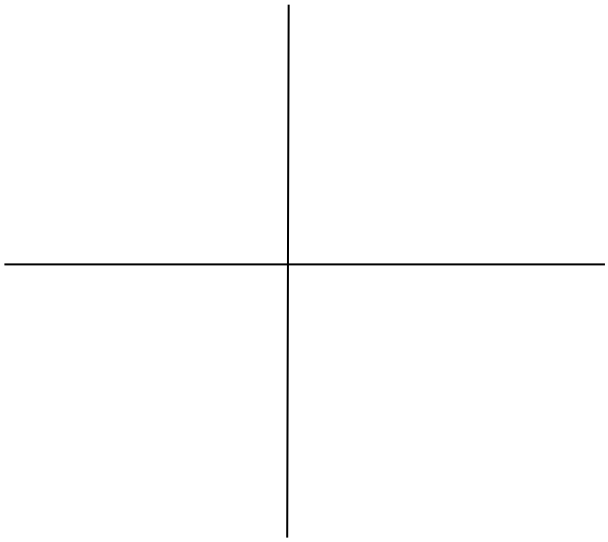
EX) Sketch the area of the region bounded by $f(x) = x^2 + 2$ and $f(x) = -x + 5$.

Write an integral and use the graphing calc. to help find its area.



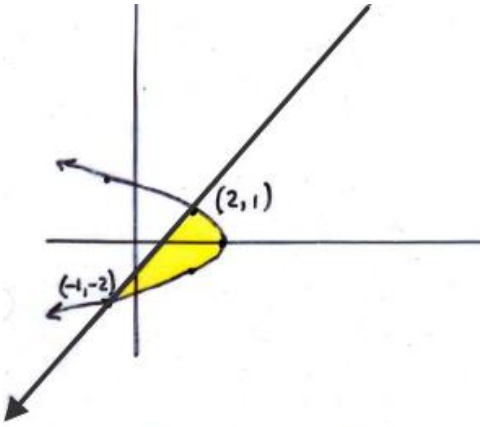
Note: it sometimes necessary to write more than one integral to follow the "Top Curve – Bottom Curve" requirement

EX) Sketch the area of the region bounded by $y = 3x^3 - x^2 - 10x$ and $y = -x^2 + 2x$. Write an integral expression and use the graphing calc. to help find its area.



When NON-Functions are used:

EX) Sketch the area of the region bounded by $y = x - 1$ and $x = 3 - y^2$. Write an integral expression and use the graphing calc. to help find its area.



Note: Since "TOP – BOTTOM" will not work, use "RIGHT-LEFT" instead. Every part of the integral must now be in terms of ***y*-values**. To do this, solve all equations for *x* in terms of *y*. Remember, that $g(y)$ means x ! Lastly use the *y*-coordinates of the points of intersection for the limits of integration.

$$A = \int_c^d \left[f(y) - g(y) \right] dy \quad \text{where } f(y) \geq g(y)$$

$$\int_{-2}^{1} \left(3 - y^2\right) - \left(y + 1\right) dy$$

Note: when using the graphing calculator to evaluate this integral, using x instead of y makes no difference in the answer(because FTC would yield the same value).

EX) Sketch the area of the region bounded by $y = x^2$, $y = 2$, $x = 4$ and $y = 0$.
Write an integral expression and use the graphing calc. to help find its area.