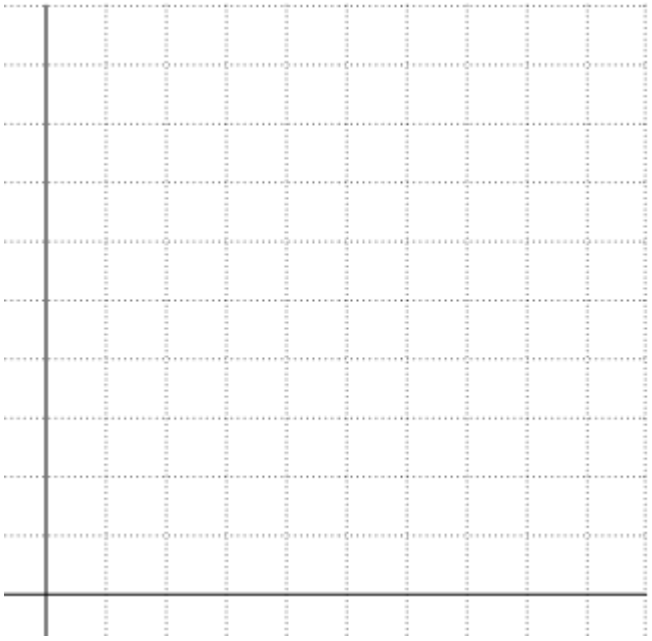
Area of a Region(left, right, midpoint and trapezoidal sums)

w-up: Graph $f(x)=\frac{1}{2}x^2+1$ on graph paper over the interval [0, 4]. Use this graph to estimate the area between the curve and the x-axis.



Left, Right, and Midpoints Sums are methods of using rectangles to estimate area under a curve when not using graph paper estimation. Decide how many rectangles to use(the more you use the more accurate the answer will be) and divide that number into the *interval length*. This will be the common width for each rectangle and often referred to as Δx. The **height** of each rectangle will vary based on the curve of the function you are using.

Left Sum: evaluating the **left** endpoint of each subinterval to determine the height of each rectangle. Note: this will be the underestimate(called ***lower sum***) for increasing functions and the overestimate(called ***upper sum***) for decreasing functions.

EX: Find the left sum for $f(x)=\frac{1}{2}x^2+1$ over [0,4] using 4 rectangles(n = 4)

Right Sum: evaluating the **right** endpoint of each subinterval to determine the height of each rectangle. Note: this will be the overestimate(called ***upper sum***) for increasing functions and the underestimate(called ***lower sum***) for decreasing functions.

EX: Find the right sum for $f(x)=\frac{1}{2}x^2+1$ over [0,4] using 4 rectangles(n = 4)

Midpoint Sum: evaluating the **midpoint** of each subinterval to determine the height of each rectangle.

EX: Find the midpoint sum for $f(x)=\frac{1}{2}x^2+1$ over [0,4] using 4 rectangles(n = 4)

The most accurate method of estimating area under a curve is using trapezoids instead of rectangles and is called the trapezoidal sum. Note: the area formula for a trapezoid is $A=\frac{(b_1+b_2)\bullet h}{2}$

Trapezoidal Sum: evaluating the **left** and **right** endpoint of each subinterval to determine the *bases* of a trapezoid. The height of the trapezoid is actually Δx. Note: this will be an overestimate when the graph is concave up and an underestimate when the graph is concave down.

EX: Find the trapezoidal sum for $f(x)=\frac{1}{2}x^2+1$ over [0,4] using 4 trapezoids(n = 4)

AP Application Problems Using Estimation of Area Under a Curve

When estimating area, it is NOT necessary to have Δx be the same value if only select *y*-values are known.

EX) Given the following data table and graph, estimate the area under the function over the interval [0, 8] using a left, right and trapezoid sum.

0	1	3	5	6	8
5	10	20	50	50	60