Find the derivative of each function.

1a) $s(t) = t^3 + 5t^2 - 3t + 8$ 1b) $f(t) = -2t^2 + 3t - 6$ 1c) $y = \frac{1}{x^5}$ 1d) $f(x) = \sqrt[5]{x}$

$s'(t) = 3t^2 + 10t - 3$

2a) $y = \frac{\pi}{2}\sin\theta - \cos\theta$ 2b) $y = x^2 - \frac{1}{2}\cos x$ 2c) $y = \frac{1}{x} - 3\sin x$ 2d) $y = \frac{6}{(5x)^3}$

$y' = \frac{\pi}{2}\cos\theta + \sin\theta$

3a) $f(x) = \frac{4x^3 + 3x^2}{x}$ 3b) $f(x) = \frac{x^3 - 3x^2 + 4}{x^2}$ 3c) $y = x(x^2 + 1)$ 3d) $f(x) = \sqrt{x} - 6\sqrt[3]{x}$

$= 4x^2 + 3x$

$f'(x) = 8x + 3$

Find the slope of the graph of the function at the given point and use it to write the equation of the tangent line to the graph at that point.

4a) $y = x^4 - 3x^2 + 2$ at (1, 0) 4b) $f(x) = \frac{2}{\sqrt[4]{x^3}}$ at (1, 2)

$y = x^4 - 3x^2 + 2$

$y' = 4x^3 - 6x$

At $(1, 0)$: $y' = 4(1)^3 - 6(1) = -2$

Tangent line: $y - 0 = -2(x - 1)$

Determine the point(s) at which the graph of the function has a horizontal tangent line.

5a) $y = x^4 - 2x^2 + 3$ 5b) $y = x^3 + x^2$ 5c) $y = \dfrac{1}{x^2}$

$$y = x^4 - 2x^2 + 3$$
$$y' = 4x^3 - 4x$$
$$= 4x(x^2 - 1)$$
$$= 4x(x - 1)(x + 1)$$
$$y' = 0 \Rightarrow x = 0, \pm 1$$

Horizontal tangents: $(0, 3), (1, 2), (-1, 2)$

5d) $y = x + \sin x, \quad 0 \le x < 2\pi$

Find *k(a constant)*, such that the line is tangent to the graph of the function.

6a) $y = 5x - 4$ tangent to $f(x) = x^2 - kx$

$x^2 - kx = 5x - 4$ **Function intersects the tangent line at the same x-value**

$2x - k = 5$ **Derivative will equal 5 at the x-value**

So, $k = 2x - 5$ and

$x^2 - (2x - 5)x = 5x - 4 \Rightarrow -x^2 = -4 \Rightarrow x = \pm 2.$

For $x = 2, k = -1$ and for $x = -2, k = -9.$

6b) $y = -\dfrac{3}{4}x + 3$ tangent to $f(x) = \dfrac{k}{x}$

7) AP MULTIPLE CHOICE EXAMPLES

1) If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$

(A) -6 (B) -3 (C) 3 (D) 6 (E) 8

2) If $f(x) = x + \sin x$, then $f'(x) =$

(A) $1 + \cos x$ (B) $1 - \cos x$ (C) $\cos x$

(D) $\sin x - x\cos x$ (E) $\sin x + x\cos x$

3) If $f(x) = \sin x$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

4) If the line $3x - 4y = 0$ is tangent in the first quadrant to the curve $y = x^3 + k$, then k is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) $-\frac{1}{8}$ (E) $-\frac{1}{2}$

5) If $f(x) = e^x$, which of the following is equal to $f'(e)$?

(A) $\lim_{h \to 0} \frac{e^{x+h}}{h}$ (B) $\lim_{h \to 0} \frac{e^{x+h} - e^e}{h}$ (C) $\lim_{h \to 0} \frac{e^{e+h} - e}{h}$

(D) $\lim_{h \to 0} \frac{e^{x+h} - 1}{h}$ (E) $\lim_{h \to 0} \frac{e^{e+h} - e^e}{h}$