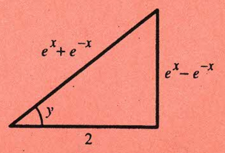
81. As k increases, the time required for the object to reach the ground increases.

82. Let $y = \arcsin(\tanh x)$. Then,

$$\sin y = \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \text{ and}$$



$$\tan y = \frac{e^x - e^{-x}}{2} = \sinh x.$$

Thus, $y = \arctan(\sinh x)$. Therefore,

$$\arctan(\sinh x) = \arcsin(\tanh x).$$

83. $y = \cosh x = \dfrac{e^x + e^{-x}}{2}$

$$y' = \frac{e^x - e^{-x}}{2} = \sinh x$$

84.
$$y = \operatorname{sech}^{-1} x$$
$$\operatorname{sech} y = x$$
$$-(\operatorname{sech} y)(\tanh y)y' = 1$$
$$y' = \frac{-1}{(\operatorname{sech} y)(\tanh y)} = \frac{-1}{(\operatorname{sech} y)\sqrt{1 - \operatorname{sech}^2 y}} = \frac{-1}{x\sqrt{1 - x^2}}$$

85.
$$y = \cosh^{-1} x$$
$$\cosh y = x$$
$$(\sinh y)(y') = 1$$
$$y' = \frac{1}{\sinh y} = \frac{1}{\sqrt{\cosh^2 y - 1}} = \frac{1}{\sqrt{x^2 - 1}}$$

86.
$$y = \sinh^{-1} x$$
$$\sinh y = x$$
$$(\cosh y)y' = 1$$
$$y' = \frac{1}{\cosh y} = \frac{1}{\sqrt{\sinh^2 y + 1}} = \frac{1}{\sqrt{x^2 + 1}}$$

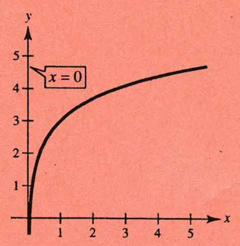
87. $y = \operatorname{sech} x = \dfrac{2}{e^x + e^{-x}}$

$$y' = -2(e^x + e^{-x})^{-2}(e^x - e^{-x}) = \left(\frac{-2}{e^x + e^{-x}}\right)\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right) = -\operatorname{sech} x \tanh x$$

Review Exercises for Chapter 5

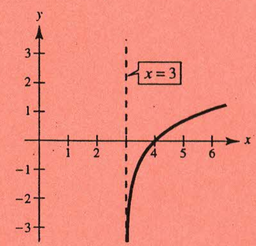
1. $f(x) = \ln x + 3$

Vertical shift 3 units upward
Vertical asymptote: $x = 0$



2. $f(x) = \ln(x - 3)$

Horizontal shift 3 units to the right
Vertical asymptote: $x = 3$



3. $\ln \sqrt[5]{\dfrac{4x^2 - 1}{4x^2 + 1}} = \dfrac{1}{5} \ln \dfrac{(2x - 1)(2x + 1)}{4x^2 + 1} = \dfrac{1}{5}[\ln(2x - 1) + \ln(2x + 1) - \ln(4x^2 + 1)]$

4. $\ln[(x^2 + 1)(x - 1)] = \ln(x^2 + 1) + \ln(x - 1)$

5. $\ln 3 + \dfrac{1}{3}\ln(4 - x^2) - \ln x = \ln 3 + \ln \sqrt[3]{4 - x^2} - \ln x = \ln\left(\dfrac{3\sqrt[3]{4 - x^2}}{x}\right)$

6. $3[\ln x - 2\ln(x^2+1)] + 2\ln 5 = 3\ln x - 6\ln(x^2+1) + \ln 5^2 = \ln x^3 - \ln(x^2+1)^6 + \ln 25 = \ln\left[\dfrac{25x^3}{(x^2+1)^6}\right]$

7. False; the domain of $f(x) = \ln x$ is the set of all **positive** real numbers.

8. False

$\ln x + \ln y = \ln(xy) \neq \ln(x+y)$

9. $\ln\sqrt{x+1} = 2$

$\sqrt{x+1} = e^2$

$x + 1 = e^4$

$x = e^4 - 1 \approx 53.598$

10. $\ln x + \ln(x-3) = 0$

$\ln x(x-3) = 0$

$x(x-3) = e^0$

$x^2 - 3x - 1 = 0$

$x = \dfrac{3 \pm \sqrt{13}}{2}$

$x = \dfrac{3+\sqrt{13}}{2}$ only since $\dfrac{3-\sqrt{13}}{2} < 0$.

11. $g(x) = \ln\sqrt{x} = \dfrac{1}{2}\ln x$

$g'(x) = \dfrac{1}{2x}$

12. $h(x) = \ln\dfrac{x(x-1)}{x-2} = \ln x + \ln(x-1) - \ln(x-2)$

$h'(x) = \dfrac{1}{x} + \dfrac{1}{x-1} - \dfrac{1}{x-2} = \dfrac{x^2-4x+2}{x^3-3x^2+2x}$

13. $f(x) = x\sqrt{\ln x}$

$f'(x) = \left(\dfrac{x}{2}\right)(\ln x)^{-1/2}\left(\dfrac{1}{x}\right) + \sqrt{\ln x}$

$= \dfrac{1}{2\sqrt{\ln x}} + \sqrt{\ln x} = \dfrac{1+2\ln x}{2\sqrt{\ln x}}$

14. $f(x) = \ln[x(x^2-2)^{2/3}] = \ln x + \dfrac{2}{3}\ln(x^2-2)$

$f'(x) = \dfrac{1}{x} + \dfrac{2}{3}\left(\dfrac{2x}{x^2-2}\right) = \dfrac{7x^2-6}{3x^3-6x}$

15. $y = \dfrac{1}{b^2}\left[\ln(a+bx) + \dfrac{a}{a+bx}\right]$

$\dfrac{dy}{dx} = \dfrac{1}{b^2}\left[\dfrac{b}{a+bx} - \dfrac{ab}{(a+bx)^2}\right] = \dfrac{x}{(a+bx)^2}$

16. $y = \dfrac{1}{b^2}[a + bx - a\ln(a+bx)]$

$\dfrac{dy}{dx} = \dfrac{1}{b^2}\left(b - \dfrac{ab}{a+bx}\right) = \dfrac{x}{a+bx}$

17. $y = -\dfrac{1}{a}\ln\left(\dfrac{a+bx}{x}\right) = -\dfrac{1}{a}[\ln(a+bx) - \ln x]$

$\dfrac{dy}{dx} = -\dfrac{1}{a}\left(\dfrac{b}{a+bx} - \dfrac{1}{x}\right) = \dfrac{1}{x(a+bx)}$

18. $y = -\dfrac{1}{ax} + \dfrac{b}{a^2}\ln\dfrac{a+bx}{x}$

$= -\dfrac{1}{ax} + \dfrac{b}{a^2}[\ln(a+bx) - \ln x]$

$\dfrac{dy}{dx} = -\dfrac{1}{a}\left(-\dfrac{1}{x^2}\right) + \dfrac{b}{a^2}\left[\dfrac{b}{a+bx} - \dfrac{1}{x}\right]$

$= \dfrac{1}{ax^2} + \dfrac{b}{a^2}\left[\dfrac{-a}{x(a+bx)}\right] = \dfrac{1}{ax^2} - \dfrac{b}{ax(a+bx)}$

$= \dfrac{(a+bx) - bx}{ax^2(a+bx)} = \dfrac{1}{x^2(a+bx)}$

19. $u = 7x - 2,\ du = 7dx$

$$\int \frac{1}{7x-2}\,dx = \frac{1}{7}\int \frac{1}{7x-2}(7)\,dx = \frac{1}{7}\ln|7x-2| + C$$

20. $u = x^2 - 1,\ du = 2x\,dx$

$$\int \frac{x}{x^2-1}\,dx = \frac{1}{2}\int \frac{2x}{x^2-1}\,dx = \frac{1}{2}\ln|x^2-1| + C$$

21. $$\int \frac{\sin x}{1+\cos x}\,dx = -\int \frac{-\sin x}{1+\cos x}\,dx = -\ln|1+\cos x| + C$$

22. $u = \ln x,\ du = \frac{1}{x}\,dx$

$$\int \frac{\ln\sqrt{x}}{x}\,dx = \frac{1}{2}\int (\ln x)\left(\frac{1}{x}\right)dx = \frac{1}{4}(\ln x)^2 + C$$

23. $$\int_1^4 \frac{x+1}{x}\,dx = \int_1^4 \left(1 + \frac{1}{x}\right)dx = \Big[x + \ln|x|\Big]_1^4 = 3 + \ln 4$$

24. $$\int_1^e \frac{\ln x}{x}\,dx = \int_1^e (\ln x)^1\left(\frac{1}{x}\right)dx = \left[\frac{1}{2}(\ln x)^2\right]_1^e = \frac{1}{2}$$

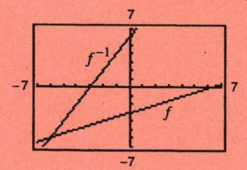
25. $$\int_0^{\pi/3} \sec\theta\,d\theta = \Big[\ln|\sec\theta + \tan\theta|\Big]_0^{\pi/3} = \ln\left(2 + \sqrt{3}\right)$$

26. $$\int_0^{\pi/4} \tan\left(\frac{\pi}{4} - x\right)dx = \left[\ln\left|\cos\left(\frac{\pi}{4} - x\right)\right|\right]_0^{\pi/4} = 0 - \ln\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}\ln 2$$

27. (a)

$$f(x) = \tfrac{1}{2}x - 3$$
$$y = \tfrac{1}{2}x - 3$$
$$2(y+3) = x$$
$$2(x+3) = y$$
$$f^{-1}(x) = 2x + 6$$

(b)



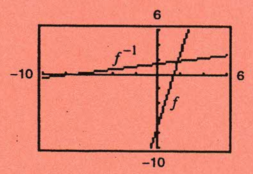
(c) $f^{-1}(f(x)) = f^{-1}\left(\tfrac{1}{2}x - 3\right) = 2\left(\tfrac{1}{2}x - 3\right) + 6 = x$

$f(f^{-1}(x)) = f(2x+6) = \tfrac{1}{2}(2x+6) - 3 = x$

28. (a)

$$f(x) = 5x - 7$$
$$y = 5x - 7$$
$$\frac{y+7}{5} = x$$
$$\frac{x+7}{5} = y$$
$$f^{-1}(x) = \frac{x+7}{5}$$

(b)



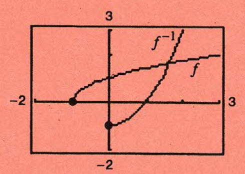
(c) $f^{-1}(f(x)) = f^{-1}(5x-7) = \dfrac{(5x-7)+7}{5} = x$

$f(f^{-1}(x)) = f\left(\dfrac{x+7}{5}\right) = 5\left(\dfrac{x+7}{5}\right) - 7 = x$

29. (a)

$$f(x) = \sqrt{x+1}$$
$$y = \sqrt{x+1}$$
$$y^2 - 1 = x$$
$$x^2 - 1 = y$$
$$f^{-1}(x) = x^2 - 1,\ x \ge 0$$

(b)



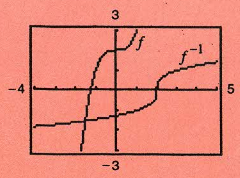
(c) $f^{-1}(f(x)) = f^{-1}\left(\sqrt{x+1}\right) = \sqrt{(x^2-1)^2} - 1 = x$

$f(f^{-1}(x)) = f(x^2 - 1) = \sqrt{(x^2-1)+1}$

$= \sqrt{x^2} = x$ for $x \ge 0$.

30. (a)
$$f(x) = x^3 + 2$$
$$y = x^3 + 2$$
$$\sqrt[3]{y - 2} = x$$
$$\sqrt[3]{x - 2} = y$$
$$f^{-1}(x) = \sqrt[3]{x - 2}$$

(b)

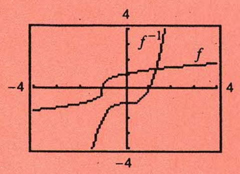


(c) $f^{-1}(f(x)) = f^{-1}(x^3 + 2) = \sqrt[3]{(x^3 + 2) - 2} = x$

$f(f^{-1}(x)) = f\left(\sqrt[3]{x - 2}\right) = \left(\sqrt[3]{x - 2}\right)^3 + 2 = x$

31. (a)
$$f(x) = \sqrt[3]{x + 1}$$
$$y = \sqrt[3]{x + 1}$$
$$y^3 - 1 = x$$
$$x^3 - 1 = y$$
$$f^{-1}(x) = x^3 - 1$$

(b)

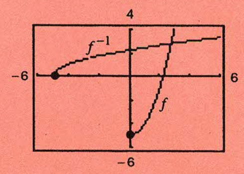


(c) $f^{-1}(f(x)) = f^{-1}\left(\sqrt[3]{x + 1}\right) = \left(\sqrt[3]{x + 1}\right)^3 - 1 = x$

$f(f^{-1}(x)) = f(x^3 - 1) = \sqrt[3]{(x^3 - 1) + 1} = x$

32. (a)
$$f(x) = x^2 - 5, x \geq 0$$
$$y = x^2 - 5$$
$$\sqrt{y + 5} = x$$
$$\sqrt{x + 5} = y$$
$$f^{-1}(x) = \sqrt{x + 5}$$

(b)

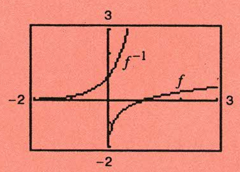


(c) $f^{-1}(f(x)) = f^{-1}(x^2 - 5) = \sqrt{(x^2 - 5) + 5} = x$ for $x \geq 0$.

$f(f^{-1}(x)) = f\left(\sqrt{x + 5}\right) = \left(\sqrt{x + 5}\right)^2 - 5 = x$

33. (a)
$$f(x) = \ln \sqrt{x}$$
$$y = \ln \sqrt{x}$$
$$e^y = \sqrt{x}$$
$$e^{2y} = x$$
$$e^{2x} = y$$
$$f^{-1}(x) = e^{2x}$$

(b)

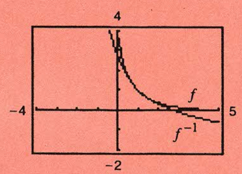


(c) $f^{-1}(f(x)) = f^{-1}\left(\ln \sqrt{x}\right) = e^{2 \ln \sqrt{x}} = e^{\ln x} = x$

$f(f^{-1}(x)) = f(e^{2x}) = \ln \sqrt{e^{2x}} = \ln e^x = x$

34. (a)
$$f(x) = e^{1-x}$$
$$y = e^{1-x}$$
$$\ln y = 1 - x$$
$$x = 1 - \ln y$$
$$y = 1 - \ln x$$
$$f^{-1}(x) = 1 - \ln x$$

(b)

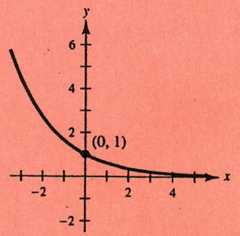


(c) $f^{-1}(f(x)) = f^{-1}\left(e^{1-x}\right) = 1 - \ln(e^{1-x})$

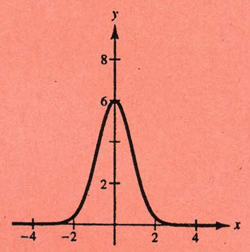
$= 1 - (1 - x) = x$

$f(f^{-1}(x)) = f(1 - \ln x) = e^{1-(1-\ln x)} = e^{\ln x} = x$

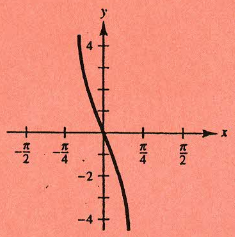
35. $y = e^{-x/2}$



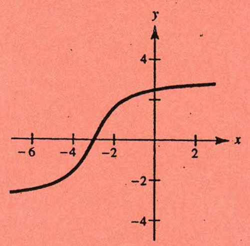
36. $g(x) = 6(2^{-x^2})$



37. $h(x) = -3\arcsin(2x)$



38. $f(x) = 2\arctan(x + 3)$



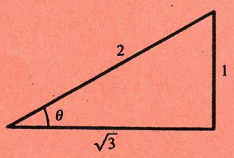
39. (a) Let $\theta = \arcsin \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

$\sin\left(\arcsin \frac{1}{2}\right) = \sin\theta = \frac{1}{2}.$

(b) Let $\theta = \arcsin \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

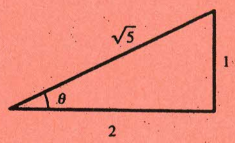


$\cos\left(\arcsin \frac{1}{2}\right) = \cos\theta = \frac{\sqrt{3}}{2}.$

40. (a) Let $\theta = \operatorname{arccot} 2$

$\cot\theta = 2$

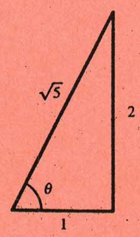
$\tan(\operatorname{arccot} 2) = \tan\theta = \frac{1}{2}.$



(b) Let $\theta = \operatorname{arcsec}\sqrt{5}$

$\sec\theta = \sqrt{5}$

$\cos\left(\operatorname{arcsec}\sqrt{5}\right) = \cos\theta = \frac{1}{\sqrt{5}}.$



41. $f(x) = \ln(e^{-x^2}) = -x^2$

$f'(x) = -2x$

42. $g(x) = \ln\left(\frac{e^x}{1 + e^x}\right)$

$= \ln e^x - \ln(1 + e^x) = x - \ln(1 + e^x)$

$g'(x) = 1 - \frac{e^x}{1 + e^x} = \frac{1}{1 + e^x}$

43. $g(t) = t^2 e^t$

$g'(x) = t^2 e^t + 2te^t = te^t(t + 2)$

44. $h(z) = e^{-z^2/2}$

$h'(z) = -ze^{-z^2/2}$

45. $y = \sqrt{e^{2x} + e^{-2x}}$

$y' = \frac{1}{2}(e^{2x} + e^{-2x})^{-1/2}(2e^{2x} - 2e^{-2x}) = \frac{e^{2x} - e^{-2x}}{\sqrt{e^{2x} + e^{-2x}}}$

46. $y = x^{2x+1}$

$\ln y = (2x + 1)\ln x$

$\frac{y'}{y} = \frac{2x + 1}{x} + 2\ln x$

$y' = y\left(\frac{2x + 1}{x} + 2\ln x\right) = x^{2x+1}\left(\frac{2x + 1}{x} + 2\ln x\right)$

47. $f(x) = 3^{x-1}$

$f'(x) = 3^{x-1} \ln 3$

48. $f(x) = 4^x e^x$

$f'(x) = 4^x e^x + (\ln 4) 4^x e^x = 4^x e^x (1 + \ln 4)$

49. $g(x) = \dfrac{x^2}{e^x}$

$g'(x) = \dfrac{e^x(2x) - x^2 e^x}{e^{2x}} = \dfrac{x(2-x)}{e^x}$

50. $f(\theta) = \dfrac{1}{2} e^{\sin 2\theta}$

$f'(\theta) = \cos 2\theta \, e^{\sin 2\theta}$

51. $y = \tan(\arcsin x) = \dfrac{x}{\sqrt{1-x^2}}$

$y' = \dfrac{(1-x^2)^{1/2} + x^2(1-x^2)^{-1/2}}{1-x^2} = (1-x^2)^{-3/2}$

52. $y = \arctan(x^2 - 1)$

$y' = \dfrac{2x}{1 + (x^2-1)^2} = \dfrac{2x}{x^4 - 2x^2 + 2}$

53. $y = x \operatorname{arcsec} x$

$y' = \dfrac{x}{|x|\sqrt{x^2-1}} + \operatorname{arcsec} x$

54. $y = \dfrac{1}{2} \arctan e^{2x}$

$y' = \dfrac{1}{2}\left(\dfrac{1}{1+e^{4x}}\right)(2e^{2x}) = \dfrac{e^{2x}}{1+e^{4x}}$

55. $y = x(\arcsin x)^2 - 2x + 2\sqrt{1-x^2} \arcsin x$

$y' = \dfrac{2x \arcsin x}{\sqrt{1-x^2}} + (\arcsin x)^2 - 2 + \dfrac{2\sqrt{1-x^2}}{\sqrt{1-x^2}} - \dfrac{2x}{\sqrt{1-x^2}} \arcsin x = (\arcsin x)^2$

56. $y = \sqrt{x^2-4} - 2 \operatorname{arcsec} \dfrac{x}{2}, \; 2 < x < 4$

$y' = \dfrac{x}{\sqrt{x^2-4}} - \dfrac{1}{(|x|/2)\sqrt{(x/2)^2 - 1}} = \dfrac{x}{\sqrt{x^2-4}} - \dfrac{4}{|x|\sqrt{x^2-4}} = \dfrac{x^2-4}{|x|\sqrt{x^2-4}} = \dfrac{\sqrt{x^2-4}}{x}$

57. $y = 2x - \cosh\sqrt{x}$

$y' = 2 - \dfrac{1}{2\sqrt{x}}(\sinh\sqrt{x}) = 2 - \dfrac{\sinh\sqrt{x}}{2\sqrt{x}}$

58. $y = x \tanh^{-1} 2x$

$y' = x\left(\dfrac{2}{1-4x^2}\right) + \tanh^{-1} 2x = \dfrac{2x}{1-4x^2} + \tanh^{-1} 2x$

59.

$$y(\ln x) + y^2 = 0$$

$$y\left(\frac{1}{x}\right) + (\ln x)\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$$

$$(2y + \ln x)\frac{dy}{dx} = \frac{-y}{x}$$

$$\frac{dy}{dx} = \frac{-y}{x(2y + \ln x)}$$

60.

$$\cos x^2 = xe^y$$

$$-2x \sin x^2 = xe^y \frac{dy}{dx} + e^y$$

$$\frac{dy}{dx} = -\frac{2x \sin x^2 + e^y}{xe^y}$$

61. (a) $y = x^a$

$y' = ax^{a-1}$

(b) $y = a^x$

$y' = (\ln a)a^x$

(c) $y = x^x$

$\ln y = x \ln x$

$\dfrac{1}{y} y' = x \cdot \dfrac{1}{x} + (1) \ln x$

$y' = y(1 + \ln x)$

$y' = x^x(1 + \ln x)$

(d) $y = a^a$

$y' = 0$

62. $10{,}000 = Pe^{(0.07)(15)}$

$P = \dfrac{10{,}000}{e^{1.05}} \approx \$3499.38$

63. $2P = Pe^{10r}$

$2 = e^{10r}$

$\ln 2 = 10r$

$r = \dfrac{\ln 2}{10} \approx 6.93\%$