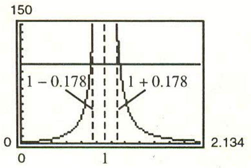
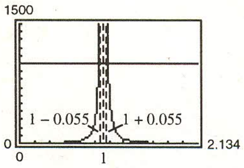
63. $f(x) = \dfrac{x + 2}{(x - 1)^2}$

(a) $\delta \approx 0.178$ for $M = 100$.



(b) $\delta \approx 0.055$ for $M = 1000$.



(c) As M increases, δ decreases.

Review Exercises for Chapter 1

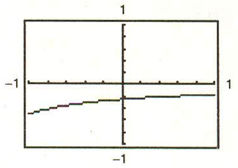
1. Calculus required. Using a graphing utility, you can estimate the length to be 8.3. Or, the length is slightly longer than the distance between the two points, 8.25.

2. Precalculus. $L = \sqrt{(9 - 1)^2 + (3 - 1)^2} \approx 8.25$

3.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	-0.26	-0.25	-0.250	-0.2499	-0.249	-0.24

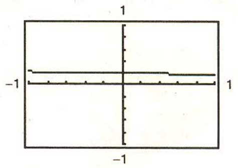
$\lim_{x \to 0} f(x) \approx -0.25$



4.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	0.202	0.2	0.2	0.2	0.2	0.198

$\lim_{x \to 0} f(x) \approx 0.2$



5. $h(x) = \dfrac{x^2 - 2x}{x}$

(a) $\lim_{x \to 0} h(x) = -2$

(b) $\lim_{x \to -1} h(x) = -3$

6. $g(x) = \dfrac{3x}{x - 2}$

(a) $\lim_{x \to 2^+} g(x) = \infty$

(b) $\lim_{x \to 0} g(x) = 0$

7. $\lim_{x \to 2} (5x - 3) = 5(2) - 3 = 7$

8. $\lim_{x \to 2} (3x + 5) = 3(2) + 5 = 11$

9. $\lim_{x \to 2} (5x - 3)(3x + 5) = [5(2) - 3][3(2) + 5]$
$= 7 \cdot 11 = 77$

10. $\lim_{x \to 2} \left(\dfrac{3x + 5}{5x - 3}\right) = \dfrac{3(2) + 5}{5(2) - 3} = \dfrac{11}{7}$

11. $\lim_{t \to 3} \dfrac{t^2 + 1}{t} = \dfrac{3^2 + 1}{3} = \dfrac{10}{3}$

12. $\lim_{t \to 3} \dfrac{t^2 - 9}{t - 3} = \lim_{t \to 3} (t + 3) = 6$

13. $\lim_{t \to -2} \dfrac{t + 2}{t^2 - 4} = \lim_{t \to -2} \dfrac{1}{t - 2} = -\dfrac{1}{4}$

14. $\lim_{x \to 0} \dfrac{\sqrt{4 + x} - 2}{x} = \lim_{x \to 0} \dfrac{\sqrt{4 + x} - 2}{x} \cdot \dfrac{\sqrt{4 + x} + 2}{\sqrt{4 + x} + 2}$
$= \lim_{x \to 0} \dfrac{1}{\sqrt{4 + x} + 2} = \dfrac{1}{4}$

15. $\lim_{x \to 0} \dfrac{[1/(x + 1)] - 1}{x} = \lim_{x \to 0} \dfrac{1 - (x + 1)}{x(x + 1)}$
$= \lim_{x \to 0} \dfrac{-1}{x + 1} = -1$

16. $$\lim_{s\to 0}\frac{\left(1/\sqrt{1+s}\right)-1}{s} = \lim_{s\to 0}\left[\frac{\left(1/\sqrt{1+s}\right)-1}{s}\cdot\frac{\left(1/\sqrt{1+s}\right)+1}{\left(1/\sqrt{1+s}\right)+1}\right]$$

$$= \lim_{s\to 0}\frac{[1/(1+s)]-1}{s\left[\left(1/\sqrt{1+s}\right)+1\right]} = \lim_{s\to 0}\frac{-1}{(1+s)\left[\left(1/\sqrt{1+s}\right)+1\right]} = -\frac{1}{2}$$

17. $$\lim_{x\to -5}\frac{x^3+125}{x+5} = \lim_{x\to -5}\frac{(x+5)(x^2-5x+25)}{x+5}$$

$$= \lim_{x\to -5}(x^2-5x+25)$$

$$= 75$$

18. $$\lim_{x\to -2}\frac{x^2-4}{x^3+8} = \lim_{x\to -2}\frac{(x+2)(x-2)}{(x+2)(x^2-2x+4)}$$

$$= \lim_{x\to -2}\frac{x-2}{x^2-2x+4}$$

$$= -\frac{4}{12} = -\frac{1}{3}$$

19. $$\lim_{x\to 0^+}\left(x-\frac{1}{x^3}\right) = -\infty$$

20. $$\lim_{x\to 2^+}\frac{1}{\sqrt[3]{x^2-4}} = \infty$$

$$\lim_{x\to 2^-}\frac{1}{\sqrt[3]{x^2-4}} = -\infty$$

Thus, $\lim_{x\to 2}\frac{1}{\sqrt[3]{x^2-4}}$ does not exist.

21. $$\lim_{\Delta x\to 0}\frac{\sin[(\pi/6)+\Delta x]-(1/2)}{\Delta x} = \lim_{\Delta x\to 0}\frac{\sin(\pi/6)\cos\Delta x+\cos(\pi/6)\sin\Delta x-(1/2)}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{1}{2}\cdot\frac{(\cos\Delta x-1)}{\Delta x} + \lim_{\Delta x\to 0}\frac{\sqrt{3}}{2}\cdot\frac{\sin\Delta x}{\Delta x}$$

$$= 0 + \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$$

22. $$\lim_{\Delta x\to 0}\frac{\cos(\pi+\Delta x)+1}{\Delta x} = \lim_{\Delta x\to 0}\frac{\cos\pi\cos\Delta x-\sin\pi\sin\Delta x+1}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\left[-\frac{(\cos\Delta x-1)}{\Delta x}\right] - \lim_{\Delta x\to 0}\left[\sin\pi\frac{\sin\Delta x}{\Delta x}\right]$$

$$= -0-(0)(1) = 0$$

23. $$\lim_{x\to -2^-}\frac{2x^2+x+1}{x+2} = -\infty$$

24. $$\lim_{x\to (1/2)^+}\frac{x}{2x-1} = \infty$$

25. $$\lim_{x\to -1^+}\frac{x+1}{x^3+1} = \lim_{x\to -1^+}\frac{1}{x^2-x+1} = \frac{1}{3}$$

26. $$\lim_{x\to -1^-}\frac{x+1}{x^4-1} = \lim_{x\to -1^-}\frac{1}{(x^2+1)(x-1)} = -\frac{1}{4}$$

27. $$\lim_{x\to 1^-}\frac{x^2+2x+1}{x-1} = -\infty$$

28. $$\lim_{x\to -1^+}\frac{x^2-2x+1}{x+1} = \infty$$

29. $$\lim_{x\to 0^+}\frac{\sin 4x}{5x} = \lim_{x\to 0^+}\left[\frac{4}{5}\left(\frac{\sin 4x}{4x}\right)\right] = \frac{4}{5}$$

30. $$\lim_{x\to 0^+}\frac{\sec x}{x} = \infty$$

31. $$\lim_{x\to 0^+}\frac{\csc 2x}{x} = \lim_{x\to 0^+}\frac{1}{x\sin 2x} = \infty$$

32. $$\lim_{x\to 0^-}\frac{\cos^2 x}{x} = -\infty$$