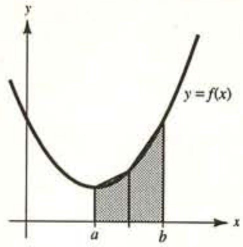
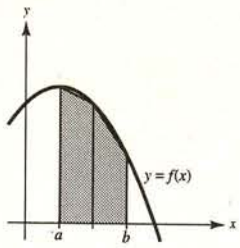
44. (a)



The Trapezoidal Rule overestimates the area if the graph of the integrand is concave up.

(b)



The Trapezoidal Rule underestimates the area if the graph of the integrand is concave down.

45. $L(x) = \int_1^x \frac{1}{t}\,dt$

(a) $L(1) = \int_1^1 \frac{1}{t}\,dt = 0$

(b) $L'(x) = \frac{1}{x}$

$L'(1) = 1$

(c) By trial and error, $x \approx 2.718$.

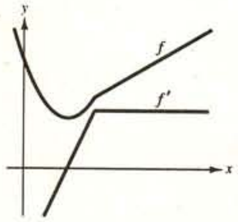
(d) $L(x_1x_2) = \int_1^{x_1x_2} \frac{1}{t}\,dt = \int_1^{x_1} \frac{1}{t}\,dt + \int_{x_1}^{x_1x_2} \frac{1}{t}\,dt$

But, this second integral is equal to $\int_1^{x_2}(1/t)\,dt$, by substitution $u = t/x_1$, $du = dt/x_1$. Hence,

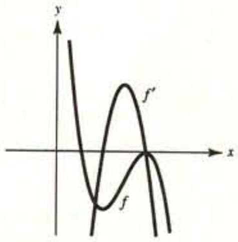
$L(x_1x_2) = L(x_1) + L(x_2).$

Review Exercises for Chapter 4

1.



2.



3. $\int (2x^2 + x - 1)\,dx = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$

4. $u = 3x$

$du = 3\,dx$

$\int \frac{2}{\sqrt[3]{3x}}\,dx = \frac{2}{3}\int (3x)^{-1/3}(3)\,dx = (3x)^{2/3} + C$

5. $\int \frac{x^3 + 1}{x^2}\,dx = \int \left(x + \frac{1}{x^2}\right)dx = \frac{1}{2}x^2 - \frac{1}{x} + C$

6. $\int \frac{x^3 - 2x^2 + 1}{x^2}\,dx = \int (x - 2 + x^{-2})\,dx$

$= \frac{1}{2}x^2 - 2x - \frac{1}{x} + C$

7. $\int (4x - 3\sin x)\,dx = 2x^2 + 3\cos x + C$

8. $\int (5\cos x - 2\sec^2 x)\,dx = 5\sin x - 2\tan x + C$

9. $f'(x) = -2x, \ (-1, 1)$

$$f(x) = \int -2x\,dx = -x^2 + C$$

When $x = -1$:

$$y = -1 + C = 1$$

$$C = 2$$

$$y = 2 - x^2$$

10. $f''(x) = 6(x - 1)$

$$f'(x) = \int 6(x - 1)\,dx = 3(x - 1)^2 + C_1$$

Since the slope of the tangent line at (2, 1) is 3, it follows that $f'(2) = 3 + C_1 = 3$ when $C_1 = 0$.

$$f'(x) = 3(x - 1)^2$$

$$f(x) = \int 3(x - 1)^2\,dx = (x - 1)^3 + C_2$$

$f(2) = 1 + C_2 = 1$ when $C_2 = 0$.

$$f(x) = (x - 1)^3$$

11. $a(t) = a$

$$v(t) = \int a\,dt = at + C_1$$

$v(0) = 0 + C_1 = 0$ when $C_1 = 0$.

$$v(t) = at$$

$$s(t) = \int at\,dt = \frac{a}{2}t^2 + C_2$$

$s(0) = 0 + C_2 = 0$ when $C_2 = 0$.

$$s(t) = \frac{a}{2}t^2$$

$$s(30) = \frac{a}{2}(30)^2 = 3600 \text{ or}$$

$$a = \frac{2(3600)}{(30)^2} = 8 \text{ ft/sec}^2.$$

$$v(30) = 8(30) = 240 \text{ ft/sec}$$

12. 45 mph = 66 ft/sec

30 mph = 44 ft/sec

$$a(t) = -a$$

$v(t) = -at + 66$ since $v(0) = 66$ ft/sec.

$s(t) = -\frac{a}{2}t^2 + 66t$ since $s(0) = 0$.

Solving the system

$$v(t) = -at + 66 = 44$$

$$s(t) = -\frac{a}{2}t^2 + 66t = 264$$

we obtain $t = 24/5$ and $a = 55/12$. We now solve $-(55/12)t + 66 = 0$ and get $t = 72/5$. Thus,

$$s\left(\frac{72}{5}\right) = -\frac{55/12}{2}\left(\frac{72}{5}\right)^2 + 66\left(\frac{72}{5}\right) \approx 475.2 \text{ ft.}$$

Stopping distance from 30 mph to rest is

$$475.2 - 264 = 211.2 \text{ ft.}$$

13. $a(t) = -32$

$$v(t) = -32t + 96$$

$$s(t) = -16t^2 + 96t$$

(a) $v(t) = -32t + 96 = 0$ when $t = 3$ sec.

(b) $s(3) = -144 + 288 = 144$ ft

(c) $v(t) = -32t + 96 = \frac{96}{2}$ when $t = \frac{3}{2}$ sec.

(d) $s\left(\frac{3}{2}\right) = -16\left(\frac{9}{4}\right) + 96\left(\frac{3}{2}\right) = 108$ ft

14. $a(t) = -9.8 \text{ m/sec}^2$

$$v(t) = -9.8t + v_0 = -9.8t + 40$$

$$s(t) = -4.9t^2 + 40t \quad (s(0) = 0)$$

(a) $v(t) = -9.8t + 40 = 0$ when $t = \frac{40}{9.8} \approx 4.08$ sec.

(b) $s(4.08) \approx 81.63$ m

(c) $v(t) = -9.8t + 40 = 20$ when $t = \frac{20}{9.8} \approx 2.04$ sec.

(d) $s(2.04) \approx 61.2$ m

15. (a) $\sum_{i=1}^{10} (2i - 1)$ (b) $\sum_{i=1}^{n} i^3$ (c) $\sum_{i=1}^{10} (4i + 2)$

16. $x_1 = 2, x_2 = -1, x_3 = 5, x_4 = 3, x_5 = 7$

(a) $\frac{1}{5}\sum_{i=1}^{5} x_i = \frac{1}{5}(2 - 1 + 5 + 3 + 7) = \frac{16}{5}$

(b) $\sum_{i=1}^{5} \frac{1}{x_i} = \frac{1}{2} - 1 + \frac{1}{5} + \frac{1}{3} + \frac{1}{7} = \frac{37}{210}$

(c) $\sum_{i=1}^{5} (2x_i - x_i^2) = [2(2) - (2)^2] + [2(-1) - (-1)^2] + [2(5) - (5)^2] + [2(3) - (3)^2] + [2(7) - (7)^2] = -56$

(d) $\sum_{i=2}^{5} (x_i - x_{i-1}) = (-1 - 2) + [5 - (-1)] + (3 - 5) + (7 - 3) = 5$

17. (a) $S = m\left(\frac{b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{2b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{3b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{4b}{4}\right)\left(\frac{b}{4}\right) = \frac{mb^2}{16}(1 + 2 + 3 + 4) = \frac{5mb^2}{8}$

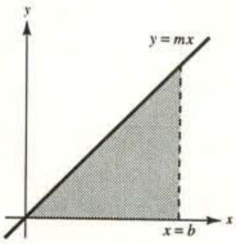
$s = m(0)\left(\frac{b}{4}\right) + m\left(\frac{b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{2b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{3b}{4}\right)\left(\frac{b}{4}\right) = \frac{mb^2}{16}(1 + 2 + 3) = \frac{3mb^2}{8}$

(b) $S(n) = \sum_{i=1}^{n} f\left(\frac{bi}{n}\right)\left(\frac{b}{n}\right) = \sum_{i=1}^{n} \left(\frac{mbi}{n}\right)\left(\frac{b}{n}\right) = m\left(\frac{b}{n}\right)^2 \sum_{i=1}^{n} i = \frac{mb^2}{n^2}\left(\frac{n(n+1)}{2}\right) = \frac{mb^2(n+1)}{2n}$

$s(n) = \sum_{i=0}^{n-1} f\left(\frac{bi}{n}\right)\left(\frac{b}{n}\right) = \sum_{i=0}^{n-1} m\left(\frac{bi}{n}\right)\left(\frac{b}{n}\right) = m\left(\frac{b}{n}\right)^2 \sum_{i=0}^{n-1} i = \frac{mb^2}{n^2}\left(\frac{(n-1)n}{2}\right) = \frac{mb^2(n-1)}{2n}$

(c) Area $= \lim_{n\to\infty} \frac{mb^2(n+1)}{2n} = \lim_{n\to\infty} \frac{mb^2(n-1)}{2n} = \frac{1}{2}mb^2 = \frac{1}{2}(b)(mb) = \frac{1}{2}$(base)(height)

(d) $\int_0^b mx\,dx = \left[\frac{1}{2}mx^2\right]_0^b = \frac{1}{2}mb^2$



18. (a) $S(n) = \sum_{i=1}^{n} f\left(1 + \frac{2i}{n}\right)\left(\frac{2}{n}\right)$

$= \sum_{i=1}^{n} \left(1 + \frac{2i}{n}\right)^3\left(\frac{2}{n}\right)$

$= \sum_{i=1}^{n} \left(1 + \frac{6i}{n} + \frac{12i^2}{n^2} + \frac{8i^3}{n^3}\right)\left(\frac{2}{n}\right)$

$= 2 + \frac{12}{n^2}\sum_{i=1}^{n} i + \frac{24}{n^3}\sum_{i=1}^{n} i^2 + \frac{16}{n^4}\sum_{i=1}^{n} i^3$

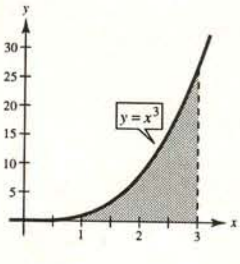
$= 2 + \frac{12}{n^2}[n(n+1)] + \frac{24}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{16}{n^4}\left[\frac{n^2(n+1)^2}{4}\right]$

$= 2 + 6\left(1 + \frac{1}{n}\right) + 4\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) + 4\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)$

Area $= \lim_{n\to\infty} S(n) = 2 + 6 + 8 + 4 = 20$

(b) $\int_1^3 x^3\,dx = \left[\frac{x^4}{4}\right]_1^3$

$= \frac{81}{4} - \frac{1}{4} = 20$



19. (a) $\int_2^6 [f(x) + g(x)]\,dx = \int_2^6 f(x)\,dx + \int_2^6 g(x)\,dx = 10 + 3 = 13$

(b) $\int_2^6 [f(x) - g(x)]\,dx = \int_2^6 f(x)\,dx - \int_2^6 g(x)\,dx = 10 - 3 = 7$

(c) $\int_2^6 [2f(x) - 3g(x)]\,dx = 2\int_2^6 f(x)\,dx - 3\int_2^6 g(x)\,dx = 2(10) - 3(3) = 11$

(d) $\int_2^6 5f(x)\,dx = 5\int_2^6 f(x)\,dx = 5(10) = 50$

20. (a) $\int_0^6 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx = 4 + (-1) = 3$

(b) $\int_6^3 f(x)\,dx = -\int_3^6 f(x)\,dx = -(-1) = 1$

(c) $\int_4^4 f(x)\,dx = 0$

(d) $\int_3^6 -10f(x)\,dx = -10\int_3^6 f(x)\,dx = -10(-1) = 10$

21. $\int (x^2+1)^3\,dx = \int (x^6 + 3x^4 + 3x^2 + 1)\,dx$

$= \frac{x^7}{7} + \frac{3}{5}x^5 + x^3 + x + C$

22. $\int \left(x + \frac{1}{x}\right)^2 dx = \int (x^2 + 2 + x^{-2})\,dx$

$= \frac{x^3}{3} + 2x - \frac{1}{x} + C$

23. $u = x^3 + 3,\ du = 3x^2\,dx$

$$\int \frac{x^2}{\sqrt{x^3+3}}\,dx = \int (x^3+3)^{-1/2}x^2\,dx = \frac{1}{3}\int (x^3+3)^{-1/2}\,3x^2\,dx = \frac{2}{3}(x^3+3)^{1/2} + C$$

24. $u = x^3 + 3,\ du = 3x^2\,dx$

$$\int x^2\sqrt{x^3+3}\,dx = \frac{1}{3}\int (x^3+3)^{1/2}\,3x^2\,dx = \frac{2}{9}(x^3+3)^{3/2} + C$$

25. $u = 1 - 3x^2,\ du = -6x\,dx$

$$\int x(1-3x^2)^4\,dx = -\frac{1}{6}\int (1-3x^2)^4(-6x\,dx) = -\frac{1}{30}(1-3x^2)^5 + C = \frac{1}{30}(3x^2-1)^5 + C$$

26. $u = x^2 + 6x - 5,\ du = (2x+6)\,dx$

$$\int \frac{x+3}{(x^2+6x-5)^2} = \frac{1}{2}\int \frac{2x+6}{(x^2+6x-5)^2}\,dx = \frac{-1}{2}(x^2+6x-5)^{-1} + C = \frac{-1}{2(x^2+6x-5)} + C$$

27. $\int \sin^3 x \cos x\,dx = \frac{1}{4}\sin^4 x + C$

28. $\int x \sin 3x^2\,dx = \frac{1}{6}\int (\sin 3x^2)(6x)\,dx = -\frac{1}{6}\cos 3x^2 + C$

29. $\int \frac{\sin\theta}{\sqrt{1-\cos\theta}}\,d\theta = \int (1-\cos\theta)^{-1/2}\sin\theta\,d\theta = 2(1-\cos\theta)^{1/2} + C = 2\sqrt{1-\cos\theta} + C$

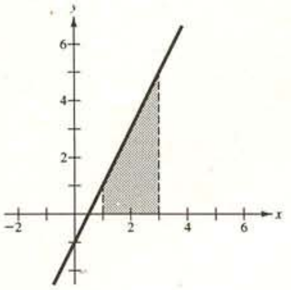
30. $\int \frac{\cos x}{\sqrt{\sin x}}\,dx = \int (\sin x)^{-1/2}\cos x\,dx = 2(\sin x)^{1/2} + C = 2\sqrt{\sin x} + C$

31. $\int \tan^n x \sec^2 x\,dx = \frac{\tan^{n+1} x}{n+1} + C,\ n \neq -1$

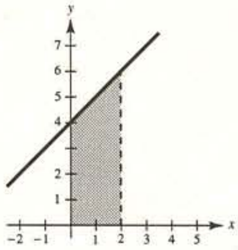
32. $\int \sec 2x \tan 2x\,dx = \frac{1}{2}\int (\sec 2x \tan 2x)(2)\,dx = \frac{1}{2}\sec 2x + C$

33. $\int (1+\sec \pi x)^2 \sec \pi x \tan \pi x\,dx = \frac{1}{\pi}\int (1+\sec\pi x)^2(\pi \sec\pi x \tan\pi x)\,dx = \frac{1}{3\pi}(1+\sec\pi x)^3 + C$

47. $\int_1^3 (2x - 1)\, dx = \left[x^2 - x\right]_1^3 = 6$



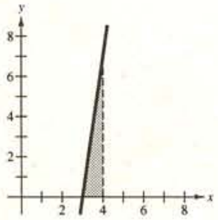
48. $\int_0^2 (x + 4)\, dx = \left[\frac{x^2}{2} + 4x\right]_0^2 = 10$



49. $\int_3^4 (x^2 - 9)\, dx = \left[\frac{x^3}{3} - 9x\right]_3^4$

$= \left(\frac{64}{3} - 36\right) - (9 - 27)$

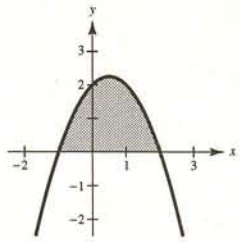
$= \frac{64}{3} - \frac{54}{3} = \frac{10}{3}$



50. $\int_{-1}^2 (-x^2 + x + 2)\, dx = \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^2$

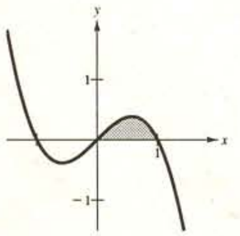
$= \left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$

$= \frac{10}{3} + \frac{7}{6} = \frac{9}{2}$



51. $\int_0^1 (x - x^3)\, dx = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$

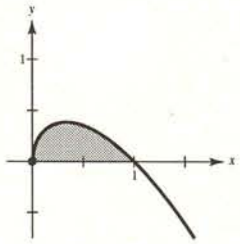
$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$



52. $\int_0^1 = \sqrt{x}(1 - x)\, dx = (x^{1/2} - x^{3/2})\, dx$

$= \left[\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2}\right]_0^1$

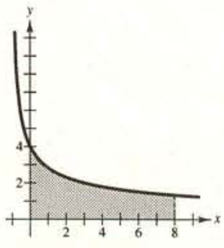
$= \frac{2}{3} - \frac{2}{5} = \frac{4}{15}$



53. $\int_0^8 \frac{4}{\sqrt{x + 1}}\, dx = 4\int_0^8 (x + 1)^{-1/2}\, dx$

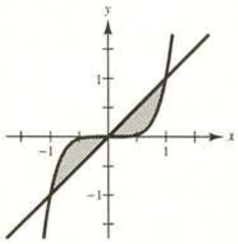
$= \left[8\sqrt{x + 1}\right]_0^8$

$= 8(3 - 1) = 16$

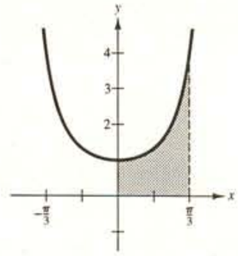


54. $\int_{-1}^{0} (x^5 - x)\, dx + \int_{0}^{1} (x - x^5)\, dx = 2\int_{0}^{1} (x - x^5)\, dx$

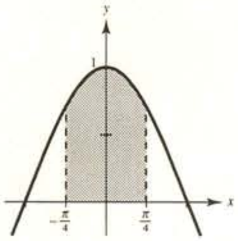
$$= 2\left[\frac{x^2}{2} - \frac{x^6}{6}\right]_0^1 = \frac{2}{3}$$



55. $\int_{0}^{\pi/3} \sec^2 x\, dx = \Big[\tan x\Big]_0^{\pi/3} = \sqrt{3}$



56. $\int_{-\pi/4}^{\pi/4} \cos x\, dx = 2\int_{0}^{\pi/4} \cos x\, dx = \Big[2 \sin x\Big]_0^{\pi/4} = \sqrt{2}$



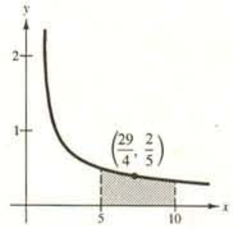
57. $\frac{1}{10 - 5}\int_{5}^{10} \frac{1}{\sqrt{x - 1}}\, dx = \left[\frac{2}{5}\sqrt{x - 1}\right]_5^{10} = \frac{2}{5}$

$$\frac{1}{\sqrt{x - 1}} = \frac{2}{5}$$

$$\sqrt{x - 1} = \frac{5}{2}$$

$$x - 1 = \frac{25}{4}$$

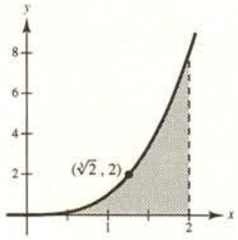
$$x = \frac{29}{4}$$



58. $\frac{1}{2 - 0}\int_{0}^{2} x^3\, dx = \left[\frac{x^4}{8}\right]_0^2 = 2$

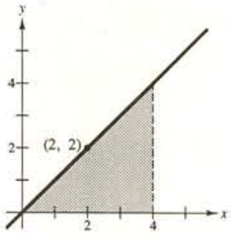
$$x^3 = 2$$

$$x = \sqrt[3]{2}$$



59. $\frac{1}{4 - 0}\int_{0}^{4} x\, dx = \left[\frac{x^2}{8}\right]_0^4 = 2$

$$x = 2$$



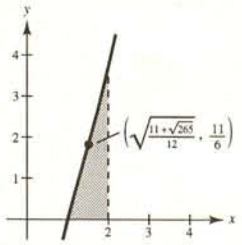
60. $\frac{1}{2 - 1}\int_{1}^{2}\left(x^2 - \frac{1}{x^2}\right) dx = \left[\frac{x^3}{3} + \frac{1}{x}\right]_1^2 = \left(\frac{8}{3} + \frac{1}{2}\right) - \left(\frac{1}{3} + 1\right) = \frac{11}{6}$

$$x^2 - \frac{1}{x^2} = \frac{11}{6}$$

$$6x^4 - 6 = 11x^2$$

$$6x^4 - 11x^2 - 6 = 0 \text{ when } x^2 = \frac{11 \pm \sqrt{265}}{12}.$$

In the interval $[1, 2]$: $x = \sqrt{\frac{11 + \sqrt{265}}{12}} \approx 1.508.$



61. Trapezoidal Rule ($n = 4$): $\int_1^2 \frac{1}{1+x^3}\,dx \approx \frac{1}{8}\left[\frac{1}{1+1^3} + \frac{2}{1+(1.25)^3} + \frac{2}{1+(1.5)^3} + \frac{2}{1+(1.75)^3} + \frac{1}{1+2^3}\right] \approx 0.257$

Simpson's Rule ($n = 4$): $\int_1^2 \frac{1}{1+x^3}\,dx \approx \frac{1}{12}\left[\frac{1}{1+1^3} + \frac{4}{1+(1.25)^3} + \frac{2}{1+(1.5)^3} + \frac{4}{1+(1.75)^3} + \frac{1}{1+2^3}\right] \approx 0.254$

Graphing utility: 0.254

62. Trapezoidal Rule ($n = 4$): $\int_0^1 \frac{x^{3/2}}{3-x^2}\,dx \approx \frac{1}{8}\left[0 + \frac{2(1/4)^{3/2}}{3-(1/4)^2} + \frac{2(1/2)^{3/2}}{3-(1/2)^2} + \frac{2(3/4)^{3/2}}{3-(3/4)^2} + \frac{1}{2}\right] \approx 0.172$

Simpson's Rule ($n = 4$): $\int_0^1 \frac{x^{3/2}}{3-x^2}\,dx \approx \frac{1}{12}\left[0 + \frac{4(1/4)^{3/2}}{3-(1/4)^2} + \frac{2(1/2)^{3/2}}{3-(1/2)^2} + \frac{4(3/4)^{3/2}}{3-(3/4)^2} + \frac{1}{2}\right] \approx 0.166$

Graphing utility: 0.166

63. (a) $C = 0.1\int_8^{20}\left[12\sin\frac{\pi(t-8)}{12}\right]dt = \left[-\frac{14.4}{\pi}\cos\frac{\pi(t-8)}{12}\right]_8^{20} = \frac{-14.4}{\pi}(-1-1) \approx \$9.17$

(b) $C = 0.1\int_{10}^{18}\left[12\sin\frac{\pi(t-8)}{12} - 6\right]dt = \left[-\frac{14.4}{\pi}\cos\frac{\pi(t-8)}{12} - 0.6t\right]_{10}^{18}$

$$= \left[\frac{-14.4}{\pi}\left(\frac{-\sqrt{3}}{2}\right) - 10.8\right] - \left[\frac{-14.4}{\pi}\left(\frac{\sqrt{3}}{2}\right) - 6\right] \approx \$3.14$$

Savings $\approx 9.17 - 3.14 = \$6.03$.

64. $\frac{1}{365}\int_0^{365} 100{,}000\left[1 + \sin\frac{2\pi(t-60)}{365}\right]dt = \frac{100{,}000}{365}\left[t - \frac{365}{2\pi}\cos\frac{2\pi(t-60)}{365}\right]_0^{365} = 100{,}000$ lbs.

65. $V = \int_0^3 0.85\sin\frac{\pi t}{3}\,dt = -\frac{3}{\pi}\left[0.85\cos\frac{\pi t}{3}\right]_0^3 = -\frac{2.55}{\pi}(-1-1) = \frac{5.1}{\pi} \approx 1.6234$ liters

66. $\int_0^2 1.75\sin\frac{\pi t}{2}\,dt = -\frac{2}{\pi}\left[1.75\cos\frac{\pi t}{2}\right]^2 = -\frac{2}{\pi}(1.75)(-1-1) = \frac{7}{\pi} \approx 2.2282$ liters

Increase is

$\frac{7}{\pi} - \frac{5.1}{\pi} = \frac{1.9}{\pi} \approx 0.6048$ liters.

67. $p = 1.20 + 0.04t$

$C = \frac{15{,}000}{M}\int_t^{t+1} p\,ds$

(a) 2000 corresponds to $t = 10$.

$$C = \frac{15{,}000}{M}\int_{10}^{11}[1.20 + 0.04t]\,dt$$
$$= \frac{15{,}000}{M}\left[1.20t + 0.02t^2\right]_{10}^{11} = \frac{24{,}300}{M}$$

(b) 2005 corresponds to $t = 15$.

$$C = \frac{15{,}000}{M}\left[1.20t + 0.02t^2\right]_{15}^{16} = \frac{27{,}300}{M}$$