31. From Exercise 23(a) we have: $V = 64\pi \text{ ft}^3$

$$\frac{1}{4}V = 16\pi$$

Disc:
$$\pi\int_{-3}^{y_0} \frac{16}{9}(9 - y^2)\,dy = 16\pi$$
$$\frac{1}{9}\int_{-3}^{y_0} (9 - y^2)\,dy = 1$$
$$\left[9y - \frac{1}{3}y^3\right]_{-3}^{y_0} = 9$$
$$\left(9y_0 - \frac{1}{3}y_0^3\right) - (-27 + 9) = 9$$
$$y_0^3 - 27y_0 - 27 = 0$$

By Newton's Method, $y_0 \approx -1.042$ and the depth of the gasoline is $3 - 1.042 = 1.958$ ft.

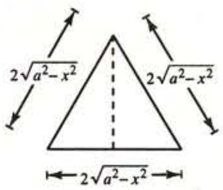
32. $A(x) = \frac{1}{2}bh = \frac{1}{2}\left(2\sqrt{a^2 - x^2}\right)\left(\sqrt{3}\sqrt{a^2 - x^2}\right)$

$= \sqrt{3}(a^2 - x^2)$

$$V = \sqrt{3}\int_{-a}^{a} (a^2 - x^2)\,dx = \sqrt{3}\left[a^2x - \frac{x^3}{3}\right]_{-a}^{a}$$
$$= \sqrt{3}\left(\frac{4a^3}{3}\right)$$

Since $\left(4\sqrt{3}\,a^3\right)/3 = 10$, we have $a^3 = \left(5\sqrt{3}\right)/2$. Thus,

$$a = \sqrt[3]{\frac{5\sqrt{3}}{2}} \approx 1.630 \text{ meters.}$$



33.
$$f(x) = \frac{4}{5}x^{5/4}$$
$$f'(x) = x^{1/4}$$
$$1 + [f'(x)]^2 = 1 + \sqrt{x}$$
$$u = 1 + \sqrt{x}$$
$$x = (u - 1)^2$$
$$dx = 2(u - 1)\,du$$
$$s = \int_0^4 \sqrt{1 + \sqrt{x}}\,dx = 2\int_1^3 \sqrt{u}(u - 1)\,du$$
$$= 2\int_1^3 (u^{3/2} - u^{1/2})\,du$$
$$= 2\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_1^3 = \frac{4}{15}\left[u^{3/2}(3u - 5)\right]_1^3$$
$$= \frac{8}{15}\left(1 + 6\sqrt{3}\right) \approx 6.076$$

34.
$$y = \frac{x^3}{6} + \frac{1}{2x}$$
$$y' = \frac{1}{2}x^2 - \frac{1}{2x^2}$$
$$1 + (y')^2 = \left(\frac{1}{2}x^2 + \frac{1}{2x^2}\right)^2$$
$$s = \int_1^3 \left(\frac{1}{2}x^2 + \frac{1}{2x^2}\right)dx = \left[\frac{1}{6}x^3 - \frac{1}{2x}\right]_1^3 = \frac{14}{3}$$

35. $y = 300\cosh\left(\frac{x}{2000}\right) - 280,\ -2000 \le x \le 2000$

$$y' = \frac{3}{20}\sinh\left(\frac{x}{2000}\right)$$
$$s = \int_{-2000}^{2000} \sqrt{1 + \left[\frac{3}{20}\sinh\left(\frac{x}{2000}\right)\right]^2}\,dx = \frac{1}{20}\int_{-2000}^{2000} \sqrt{400 + 9\sinh^2\left(\frac{x}{2000}\right)}\,dx$$

≈ 4018.2 ft (by Simpson's Rule or graphing utility)

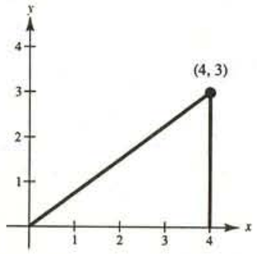
36. Since $f(x) = \tan x$ has $f'(x) = \sec^2 x$, this integral represents the length of the graph of $\tan x$ from $x = 0$ to $x = \pi/4$. This

37. $y = \frac{3}{4}x$

$$y' = \frac{3}{4}$$

$$1 + (y')^2 = \frac{25}{16}$$

$$S = 2\pi\int_0^4 \left(\frac{3}{4}x\right)\sqrt{\frac{25}{16}}\,dx = \left[\left(\frac{15\pi}{8}\right)\frac{x^2}{2}\right]_0^4 = 15\pi$$



38. $y = 2\sqrt{x}$

$$y' = \frac{1}{\sqrt{x}}$$

$$1 + (y')^2 = 1 + \frac{1}{x} = \frac{x+1}{x}$$

$$S = 2\pi\int_0^3 2\sqrt{x}\sqrt{\frac{x+1}{x}}\,dx = 4\pi\int_0^3 \sqrt{x+1}\,dx$$

$$= 4\pi\left[\left(\frac{2}{3}\right)(x+1)^{3/2}\right]_0^3 = \frac{56\pi}{3}$$

39. $F = kx$

$4 = k(1)$

$F = 4x$

$$W = \int_0^5 4x\,dx = \left[2x^2\right]_0^5$$

$= 50$ in · lb ≈ 4.167 ft · lb

40. $F = kx$

$50 = k(9) \Rightarrow k = \frac{50}{9}$

$F = \frac{50}{9}x$

$$W = \int_0^9 \frac{50}{9}x\,dx = \left[\frac{25}{9}x^2\right]_0^9$$

$= 225$ in · lb $= 18.75$ ft · lb

41. Volume of disc: $\pi\left(\frac{1}{3}\right)^2\Delta y$

Weight of disc: $62.4\pi\left(\frac{1}{3}\right)^2\Delta y$

Distance: $175 - y$

$$W = \frac{62.4\pi}{9}\int_0^{150}(175 - y)\,dy = \frac{62.4\pi}{9}\left[175y - \frac{y^2}{2}\right]_0^{150}$$

$= 104{,}000\pi$ ft · lb ≈ 163.4 ft · ton

42. We know that

$$\frac{dV}{dt} = \frac{4\text{ gal/min} - 12\text{ gal/min}}{7.481\text{ gal/ft}^3} = -\frac{8}{7.481}\text{ ft}^3\text{/min}$$

$$V = \pi r^2 h = \pi\left(\frac{1}{9}\right)h$$

$$\frac{dV}{dt} = \frac{\pi}{9}\left(\frac{dh}{dt}\right)$$

$$\frac{dh}{dt} = \frac{9}{\pi}\left(\frac{dV}{dt}\right) = \frac{9}{\pi}\left(-\frac{8}{7.481}\right) \approx -3.064\text{ ft/min.}$$

Depth of water: $-3.064t + 150$

Time to drain well: $t = \frac{150}{3.064} \approx 49$ minutes

$(49)(12) = 588$ gallons pumped

Volume of water pumped in Exercise 41: 391.7 gallons

$$\frac{391.7}{52\pi} = \frac{588}{x\pi}$$

$$x = \frac{588(52)}{391.7} \approx 78$$

Work $\approx 78\pi$ ft · ton