Find the derivative of each function.

1a) $y = \sqrt[3]{6x^2 + 1}$

$$y = \sqrt[3]{6x^2+1} = \left(6x^2+1\right)^{1/3}$$

$$y' = \frac{1}{3}\left(6x^2+1\right)^{-2/3}(12x) = \frac{4x}{\left(6x^2+1\right)^{2/3}}$$

$$= \frac{4x}{\sqrt[3]{\left(6x^2+1\right)^2}}$$

1b) $g(x) = 3(4 - 9x)^4$

2a) $y = \dfrac{1}{x-2}$

$$y = (x-2)^{-1}$$

$$y' = -1(x-2)^{-2}(1) = \frac{-1}{(x-2)^2}$$

2b) $y = \dfrac{1}{\sqrt{x+2}}$

3a) $y = \dfrac{x}{\sqrt{x^2+1}}$

$$y = \frac{x}{\sqrt{x^2+1}} = \frac{x}{\left(x^2+1\right)^{-1/2}}$$

$$y' = \frac{\left(x^2+1\right)^{1/2}(1) - x\left(\frac{1}{2}\right)\left(x^2+1\right)^{-1/2}(2x)}{\left[\left(x^2+1\right)^{1/2}\right]^2}$$

$$= \frac{\left(x^2+1\right)^{1/2} - x^2\left(x^2+1\right)^{-1/2}}{x^2+1}$$

$$= \frac{\left(x^2+1\right)^{-1/2}\left[x^2+1-x^2\right]}{x^2+1} = \frac{1}{\left(x^2+1\right)^{3/2}} = \frac{1}{\sqrt{\left(x^2+1\right)^3}}$$

3b) $y = x\sqrt{1 - x^2}$

3c) $g(x) = \left(\frac{x + 5}{x^2 + 2}\right)^2$

4a) $g(x) = 5 \tan 3x$

$g(x) = 5 \tan 3x$

$g'(x) = 15 \sec^2 3x$

4b) $f(x) = 2\cos^2 x - 1 + 4 \sin x \cos x$

4c) $h(x) = \sin 2x \cos 2x$

5a) $y = 4(\sec x)^2$

$y' = 8 \sec x \cdot \sec x \tan x$

$= 8 \sec^2 x \tan x$

5b) $f(\theta) = \frac{1}{4} \sin^2 2\theta$

5c) $f(\theta) = \tan^2 5\theta$

Evaluate the derivative of the function at the given point. Use a graphing utility to verify your result.

6a) $s(t) = \sqrt{t^2 + 6t - 2}, \quad (3, 5)$

6b) $y = 26 - \sec^3 4x, \quad (0, 25)$

Find an equation of the tangent line to the graph of at the given point and use the *derivative* feature of a graphing utility to confirm your results.

7a) $g(t) = \dfrac{3t^2}{\sqrt{t^2 + 2t - 1}}, \quad \left(\dfrac{1}{2}, \dfrac{3}{2}\right)$

8a) **Determine the point(s) in the interval** $(0, 2\pi)$ **at which the graph of** $f(x) = 2\cos x + \sin 2x$ **has a horizontal tangent.**

9) AP MULTIPLE CHOICE EXAMPLES

1) If $f(x)=(x^2-2x-1)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

2) If $f(x)=\sqrt{2x}$, then $f'(2)=$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) $\sqrt{2}$

3) If $y=2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2}=$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

4) If $y=\cos^2 3x$, then $\frac{dy}{dx}=$

(A) $-6\sin 3x\cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$

(D) $6\cos 3x$ (E) $2\sin 3x\cos 3x$

5) If $f(x)=x^{\frac{1}{3}}(x-2)^{\frac{2}{3}}$ for all x, then the domain of f' is

(A) $\{x \mid x \neq 0\}$ (B) $\{x \mid x > 0\}$ (C) $\{x \mid 0 \leq x \leq 2\}$

(D) $\{x \mid x \neq 0 \text{ and } x \neq 2\}$ (E) $\{x \mid x \text{ is a real number}\}$

6) If $f(x)=(x-1)^2 \sin x$, then $f'(0)=$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

7) If $f(x)=(2x+1)^4$, then the 4th derivative of $f(x)$ at $x=0$ is

(A) 0 (B) 24 (C) 48 (D) 240 (E) 384

8) An equation of the line tangent to the graph of $f(x)=x(1-2x)^3$ at the point $(1,-1)$ is

(A) $y=-7x+6$ (B) $y=-6x+5$ (C) $y=-2x+1$

(D) $y=2x-3$ (E) $y=7x-8$

9) If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1 (B) 0 (C) $-2\sin(2x)$ (D) $-2(\cos x + \sin x)$ (E) $2(\cos x - \sin x)$

10) AP OPEN ENDED EXAMPLES

1) The following table lists the values of functions g and h, and of their derivatives, g' and h', for the x-values -3 and 2.

x	$g(x)$	$h(x)$	$g'(x)$	$h'(x)$
-3	14	-8	-7	6
2	4	-3	3	-4

Evaluate $\frac{d}{dx}\left[g\left(h(x)\right)\right]$ at $x = 2$.

2) The following table lists the values of functions f and g, and of their derivatives, f' and g', for the x-values 1 and 2.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	4	2	3	-2
2	6	1	1	0

Let function G be defined as $G(x) = f\left(g(x)\right)$.

$G'(2) =$ ______