Find the derivative of each function.

1a) $y = \sqrt[3]{6x^2 + 1}$

$$y = \sqrt[3]{6x^2+1} = \left(6x^2+1\right)^{1/3}$$

$$y' = \frac{1}{3}\left(6x^2+1\right)^{-2/3}(12x) = \frac{4x}{\left(6x^2+1\right)^{2/3}}$$

$$= \frac{4x}{\sqrt[3]{\left(6x^2+1\right)^2}}$$

1b) $g(x) = 3(4 - 9x)^4$

$$g'(x) = 3 \cdot 4\,(4-9x)^3 \cdot (-9)$$

$$= -108\,(4-9x)^3$$

2a) $y = \dfrac{1}{x - 2}$

$$y = (x-2)^{-1}$$

$$y' = -1(x-2)^{-2}(1) = \frac{-1}{(x-2)^2}$$

2b) $y = \dfrac{1}{\sqrt{x + 2}}$

$$y = (x+2)^{-1/2}$$

$$y' = \frac{-1}{2}(x+2)^{-3/2} \cdot 1$$

$$y' = \frac{-1}{2\sqrt{(x+2)^3}}$$

3a) $y = \dfrac{x}{\sqrt{x^2 + 1}}$

$$y = \frac{x}{\sqrt{x^2+1}} = \frac{x}{\left(x^2+1\right)^{-1/2}}$$

$$y' = \frac{\left(x^2+1\right)^{1/2}(1) - x\left(\frac{1}{2}\right)\left(x^2+1\right)^{-1/2}(2x)}{\left[\left(x^2+1\right)^{1/2}\right]^2}$$

$$= \frac{\left(x^2+1\right)^{1/2} - x^2\left(x^2+1\right)^{-1/2}}{x^2+1}$$

$$= \frac{\left(x^2+1\right)^{-1/2}\left[x^2+1-x^2\right]}{x^2+1} = \frac{1}{\left(x^2+1\right)^{3/2}} = \frac{1}{\sqrt{\left(x^2+1\right)^3}}$$

3b) $y = x\sqrt{1 - x^2}$ $= x \cdot (1-x^2)^{1/2}$

$y' = x \cdot \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) + (1-x^2)^{1/2} \cdot 1$

$y' = \frac{-x^2}{\sqrt{1-x^2}} + \frac{\sqrt{1-x^2}}{1}$

$y' = \frac{-x^2 + (\sqrt{1-x^2})^2}{\sqrt{1-x^2}}$

$y' = \frac{-x^2 + 1 - x^2}{\sqrt{1-x^2}} = \frac{1-2x^2}{\sqrt{1-x^2}}$

3c) $g(x) = \left(\frac{x+5}{x^2+2}\right)^2$

$g'(x) = 2\left(\frac{x+5}{x^2+2}\right)^1 \cdot \frac{(x^2+2)\cdot 1 - (x+5)\cdot 2x}{(x^2+2)^2}$

$g'(x) = \frac{2(x+5)}{x^2+2} \cdot \frac{x^2+2-2x^2-10x}{(x^2+2)^2}$

$g'(x) = \frac{2(x+5)}{x^2+2} \cdot \frac{-x^2-10x+2}{(x^2+2)^2}$

$g'(x) = \frac{2(x+5)(-x^2-10x+2)}{(x^2+2)^3}$

4a) $g(x) = 5\tan 3x$

$g(x) = 5\tan 3x$

$g'(x) = 15\sec^2 3x$

4b) $f(x) = 2\cos^2 x - 1 + 4\sin x\cos x$

$f(x) = \cos(2x) + 2 \cdot 2\sin x\cos x$

$f(x) = \cos(2x) + 2 \cdot \sin(2x)$

$f'(x) = -\sin(2x)\cdot 2 + 2\cdot\cos(2x)\cdot 2$

$f'(x) = -2\sin(2x) + 4\cos(2x)$

4c) $h(x) = \sin 2x\cos 2x$

$h(x) = \frac{1}{2} \cdot 2\sin(2x)\cos(2x)$

$h(x) = \frac{1}{2}\sin 2(2x)$

$h(x) = \frac{1}{2}\sin(4x)$

$h'(x) = \frac{1}{2}\cos(4x)\cdot 4$

$h'(x) = 2\cos(4x)$

5a) $y = 4(\sec x)^2$

$y' = 8\sec x \cdot \sec x\tan x$

$= 8\sec^2 x\tan x$

5b) $f(\theta) = \frac{1}{4}\sin^2 2\theta$

$f(\theta) = \frac{1}{4}(\sin(2\theta))^2$

$f'(\theta) = \frac{2}{4}(\sin(2\theta))^1 \cdot \cos(2\theta)\cdot 2$

$f'(\theta) = \frac{1}{2} \cdot 2\sin(2\theta)\cos(2\theta)$

$f'(\theta) = \frac{1}{2}\sin 2(2\theta)$

$f'(\theta) = \frac{1}{2}\sin 4\theta$

5c) $f(\theta) = \tan^2 5\theta$

$f(\theta) = (\tan(5\theta))^2$

$f(\theta) = (\tan 5\theta)^2$

$f'(\theta) = 2(\tan 5\theta)^1 \cdot \sec^2(5\theta)\cdot 5$

$\boxed{f'(\theta) = 10\tan 5\theta\sec^2 5\theta}$

Evaluate the derivative of the function at the given point. Use a graphing utility to verify your result.

6a) $s(t) = \sqrt{t^2 + 6t - 2}, \quad (3, 5)$

$s(t) = (t^2+6t-2)^{1/2}$

$s'(t) = \frac{1}{2}(t^2+6t-2)^{-1/2} \cdot (2t+6)$

$s'(t) = \dfrac{2t+6}{2\sqrt{t^2+6t-2}}$

$s'(3) = \dfrac{12}{2\sqrt{25}} = \boxed{\dfrac{6}{5}}$

6b) $y = 26 - \sec^3 4x, \quad (0, 25)$

$y = 26 - (\sec(4x))^3$

$y' = 0 - 3(\sec(4x))^2 \cdot \sec 4x \tan 4x \cdot 4$

$y' = -12(\sec 4x)^3 \tan 4x$

$y'(0) = -12 \cdot (1)^3 \cdot 0$

$= \boxed{0}$

Find an equation of the tangent line to the graph of at the given point and use the *derivative* feature of a graphing utility to confirm your results.

7a) $g(t) = \dfrac{3t^2}{\sqrt{t^2 + 2t - 1}}, \quad \left(\dfrac{1}{2}, \dfrac{3}{2}\right)$

$y - \frac{3}{2} = m\left(x - \frac{1}{2}\right)$

$\boxed{y - \frac{3}{2} = -3\left(x - \frac{1}{2}\right)}$

$g(t) = \dfrac{3t^2}{(t^2+2t-1)^{1/2}}$

$g'(t) = \dfrac{(t^2+2t-1)^{1/2} \cdot 6t - 3t^2 \cdot \frac{1}{2}(t^2+2t-1)^{-1/2} \cdot (2t+2)}{\left((t^2+2t-1)^{1/2}\right)^2}$

$g'\left(\frac{1}{2}\right) = \dfrac{\frac{1}{2} \cdot 3 - \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{1}{\frac{1}{2}} \cdot 3}{\frac{1}{4}} = \dfrac{\frac{3}{2} - \frac{9}{4}}{\frac{1}{4}} \cdot \dfrac{4}{4} \quad \dfrac{6-9}{1} = -3$

8a) **Determine the point(s) in the interval** $(0, 2\pi)$ **at which the graph of** $f(x) = 2\cos x + \sin 2x$ **has a horizontal tangent.**

slope of ZERO

$f'(x) = -2\sin x + \cos 2x \cdot (2)$

$0 = -2\sin x + 2\cos 2x$

$0 = -2\sin x + 2(1 - 2\sin^2 x)$ BY IDENTITY

$4\sin^2 x + 2\sin x - 2 = 0$ ← DIVIDE BY 2

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad x = \frac{3\pi}{2}$

$\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{2}\right)$

$\left(\frac{5\pi}{6}, \frac{-3\sqrt{3}}{2}\right)$

$\left(\frac{3\pi}{2}, 0\right)$

9) AP MULTIPLE CHOICE EXAMPLES

1) If $f(x)=(x^2-2x-1)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

$f'(x)=\frac{2}{3}(x^2-2x-1)^{-1/3}\cdot(2x-2)$ $f'(0)=\frac{2}{3}(-1)^{-1/3}\cdot(-2)$ $f'(0)=\frac{2}{3}\cdot\frac{1}{\sqrt[3]{-1}}\cdot{-2}=\frac{-4}{-3}$

2) If $f(x)=\sqrt{2x}$, then $f'(2)=$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) $\sqrt{2}$

$f(x)=(2x)^{1/2}$ $f'(x)=\frac{1}{\cancel{2}}(2x)^{-1/2}\cdot\cancel{2}$ $f'(2)=\frac{1}{\sqrt{2\cdot2}}=\frac{1}{\sqrt{4}}=\frac{1}{2}$

$f'(x)=\frac{1}{\sqrt{2x}}$

3) If $y=2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2}=$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

$y'=2(-\sin(\frac{x}{2}))\cdot\frac{1}{2}$ $y''=-\cos(\frac{x}{2})\cdot\frac{1}{2}$

$y'=-\sin(\frac{x}{2})$ $y''=-\frac{1}{2}\cos(\frac{x}{2})$

4) If $y=\cos^2 3x$, then $\frac{dy}{dx}=$

(A) $-6\sin 3x\cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$

(D) $6\cos 3x$ (E) $2\sin 3x\cos 3x$

$y=(\cos(3x))^2$

$y'=2(\cos(3x))^1\cdot(-\sin(3x))\cdot 3$

$y'=-6\sin(3x)\cos(3x)$

5) If $f(x)=x^{\frac{1}{3}}(x-2)^{\frac{2}{3}}$ for all x, then the domain of f' is

(A) $\{x \mid x \neq 0\}$ (B) $\{x \mid x>0\}$ (C) $\{x \mid 0 \leq x \leq 2\}$

(D) $\{x \mid x \neq 0 \text{ and } x \neq 2\}$ (E) $\{x \mid x \text{ is a real number}\}$

$$f'(x) = x^{1/3} \cdot \frac{2}{3}(x-2)^{-1/3} \cdot 1 + (x-2)^{2/3} \cdot \frac{1}{3}x^{-2/3}$$

$$f'(x) = \frac{2\sqrt[3]{x}}{3\sqrt[3]{x-2}} + \frac{\sqrt[3]{(x-2)^2}}{3\sqrt[3]{x^2}}$$

6) If $f(x)=(x-1)^2 \sin x$, then $f'(0)=$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

$$f'(x) = (x-1)^2 \cdot \cos x + 2(x-1)^1 \cdot 1 \cdot \sin x$$

$$f'(0) = (-1)^2 \cdot \cos 0 + 2(-1) \cdot \sin 0$$

$$f'(0) = 1 \cdot 1 - 2 \cdot 0$$

$$= 1$$

7) If $f(x)=(2x+1)^4$, then the 4th derivative of $f(x)$ at $x=0$ is

(A) 0 (B) 24 (C) 48 (D) 240 (E) 384

$$f'(x) = 4(2x+1)^3 \cdot 2$$

$$f'(x) = 8(2x+1)^3$$

$$f''(x) = 24(2x+1)^2 \cdot 2$$

$$f''(x) = 48(2x+1)^2$$

$$f^3(x) = 96(2x+1)^1 \cdot 2$$

$$f^3(x) = 192(2x+1)$$

$$f^3(x) = 384x + 192$$

$$f^4(x) = 384$$

NOTE: Since 4th deriv. has no variable remaining, nowhere to put the $x=0$

8) An equation of the line tangent to the graph of $f(x)=x(1-2x)^3$ at the point $(1,-1)$ is

(A) $y=-7x+6$ (B) $y=-6x+5$ (C) $y=-2x+1$

(D) $y=2x-3$ (E) $y=7x-8$

$$f'(x) = x \cdot 3(1-2x)^2 \cdot (-2x) + (1-2x)^3 \cdot 1$$

$$f'(1) = 1 \cdot 3(-1)^2 \cdot (-2) + (-1)^3$$

$$f'(1) = -6 - 1$$

$$f'(1) = -7$$

So... $(1,-1)$

slope: -7

$$y+1 = -7(x-1)$$

$$y = -7x + 7 - 1$$

$$y = -7x + 6$$

9) If $y=\cos^2 x-\sin^2 x$, then $y'=$

A) -1 (B) 0 (C) $-2\sin(2x)$ (D) $-2(\cos x+\sin x)$ (E) $2(\cos x-\sin x)$

$y = \cos 2x$ $\quad y' = -\sin(2x)(2)$

"by identity"

OR $y=(\cos x)^2-(\sin x)^2$

$y' = 2\cos x \cdot (-\sin x) - 2\sin x \cos x$

$y' = -2\sin x \cos x - 2\sin x\cos x$

by identity

$y' = -\sin 2x - \sin 2x$

$y' = -2\sin 2x$

AP OPEN ENDED EXAMPLES

The following table lists the values of functions g and h, and of their

values -3 and 2.

10) AP OPEN ENDED EXAMPLES

1) The following table lists the values of functions g and h, and of their derivatives, g' and h', for the x-values -3 and 2.

x	$g(x)$	$h(x)$	$g'(x)$	$h'(x)$
-3	14	-8	-7	6
2	4	-3	3	-4

Evaluate $\frac{d}{dx}\left[g\left(h(x)\right)\right]$ at $x=2$.

$\frac{d}{dx}\left(g(h(x))\right) = g'(h(x)) \cdot h'(x)$

at $x=2$ $\quad g'(h(2)) \cdot h'(2)$

$g'(-3)\cdot h'(2) = -7 \cdot -4 = \boxed{28}$

2) The following table lists the values of functions f and g, and of their derivatives, f' and g', for the x-values 1 and 2.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	4	2	3	−2
2	6	1	1	0

Let function G be defined as $G(x) = f\big(g(x)\big)$.

$G'(2) =$ 0

$G'(x) = f'(g(x)) \cdot g'(x)$

$G'(2) = f'(g(2)) \cdot g'(2)$

$G'(2) = f'(1) \cdot g'(2)$

$G'(2) = 3 \cdot 0$

$G'(2) = \boxed{0}$