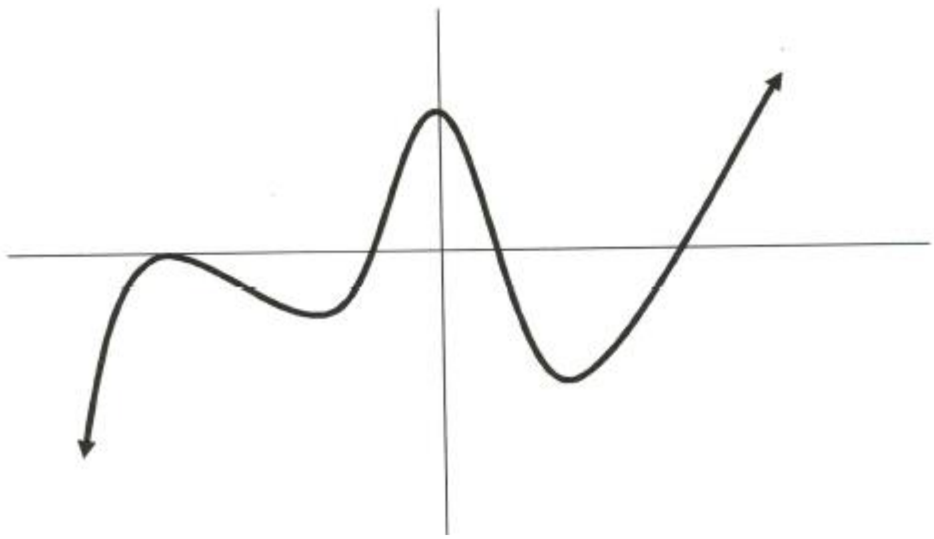
Concavity & the Second Derivative Test

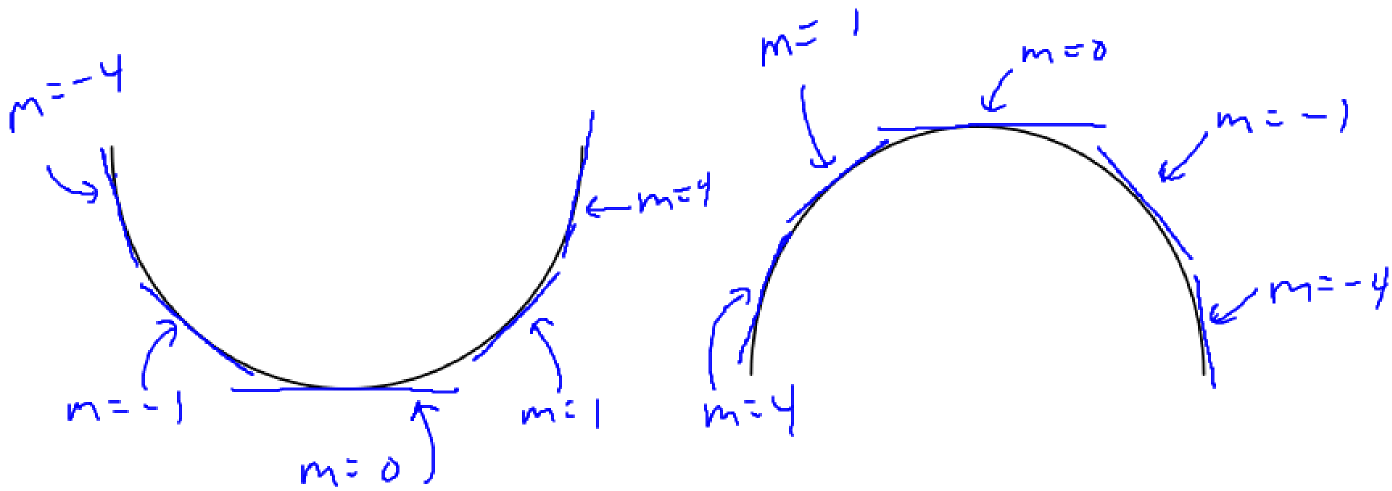
w-up: AP multiple choice #15(no calculator allowed)



Review the graph with tangent lines from Unit 4 lesson 3 notes and determine the similarities in sections of the graph where the tangent lines are ABOVE the graph and also similarities in sections of the graph where the tangent lines are BELOW the graph.

Concavity

Concave Up	Concave Down
Graph opens upward “holds water”	Graph opens downward “spills water”
Tangent Lines are BELOW the graph	Tangent Lines are ABOVE the graph



A function is considered **CONCAVE UP** where its ***slopes*** are increasing and **CONCAVE DOWN** where its ***slopes*** are decreasing.

Inflection Point: point on a function where its graph changes concavity

Note: a graph can also change concavity over an asymptote!

Remember that we use the derivative of a function to determine when the FUNCTION increases/decreases. Since concavity is determined by knowing when the SLOPES are increasing/decreasing we actually use the 1st Derivative Test ***on the derivative***(which is the functional expression for the slopes of a function)! So, actually the 2nd derivative is used to determine concavity!

Using the Second Derivative to find intervals of Upward/Downward Concavity and x -values for Inflection Points(if they occur)

1) Find x-values that make the 2^{nd} derivative zero or undefined and place them on a number line.

2) Pick any value in between these critical values and evaluate them in the 2^{nd} derivative. If ***positive***, the **slopes** of the function must be increasing thus the graph is CONCAVE UP in that entire interval. If ***negative***, the **slopes** of the function must be decreasing thus the graph is CONCAVE DOWN in that entire interval.

3) If there is a sign change(concavity change) over any DEFINED x-value, then it represents the x-value of an **inflection point**.

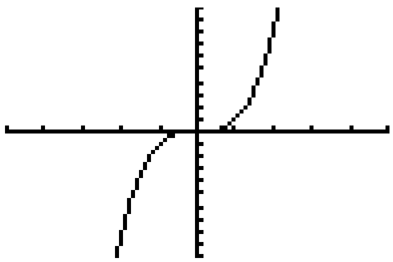
EX For each function, determine(without a calculator) intervals of concavity and identify any ordered pairs of inflection points.

A) $f(x)=\frac{1}{3}x^3-x^2$ B) $f(x)=2x^3-3x^2-36x+14$ C) $f(x)=(x-3)^{1/3}$

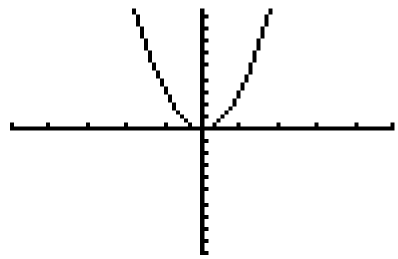
READING A DERIVATIVE GRAPH TO DETERMINE TRAITS OF ORIGINAL FUNCTION

Compare the graphs of a function and its derivative below.

$f(x)=x^3$



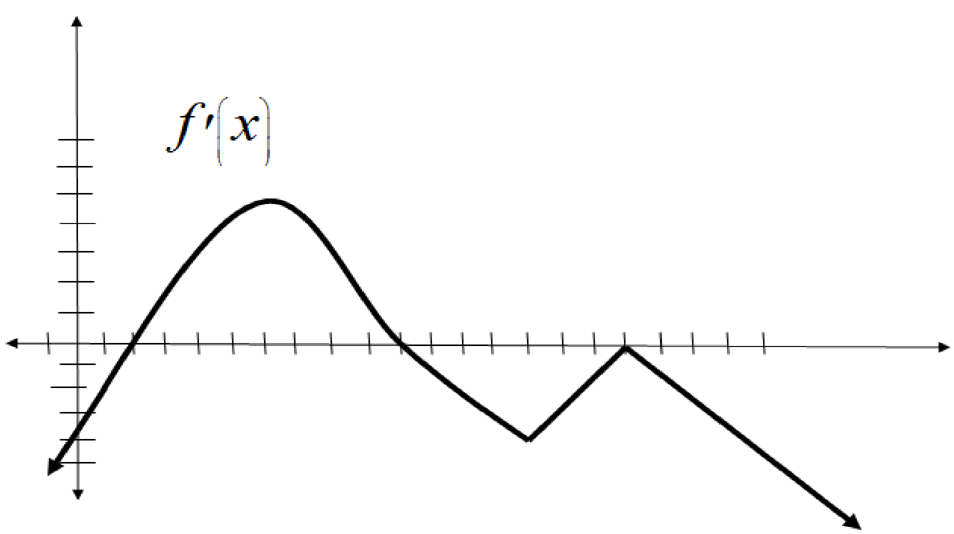
$f'(x)=3x^2$



When reading a derivative graph($f'(x)$):

Intervals where the *derivative* graph is INCREASING are intervals where the original function is CONCAVE UP and intervals where the *derivative* graph is DECREASING are intervals where the original function is CONCAVE DOWN. So, x-values of a relative min/max on the ***derivative graph*** must be x-values of **inflection points** on the original function!

EX) Use the graph of the **derivative** below to list intervals of concavity **on *f*(*x*)** and identify any x-values of inflection points.



Using the 2nd Derivative to Determine Maximums/Minimums(called 2nd derivative test)

If a slope of zero occurs at an x-value on a **concave up** interval it must be a relative **MINIMUM** while if it occurs on a **concave down** interval it must be a relative **MAXIMUM**.

> **THEOREM 3.9 Second Derivative Test**
>
> Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.
>
> **1.** If $f''(c) > 0$, then f has a relative minimum at $(c, f(c))$.
>
> **2.** If $f''(c) < 0$, then f has a relative maximum at $(c, f(c))$.
>
> If $f''(c) = 0$, the test fails. That is, f may have a relative maximum at c, a relative minimum at $(c, f(c))$, or neither. In such cases, you can use the First Derivative Test.

AP EXAMPLES

#1) Given f is a continuous and differentiable function over all real numbers. If $f''(x)=0$ at x = -2 and x = 4 ONLY, $f''(x)<0$ when $x<-2$ and $x>4$ while $f''(x)>0$ when $-2<x<4$.

If $f'(-5)=0$ then a ________________ must occur at x = -5

If $f'(0)=0$ then a ________________ must occur at x = 0

#2) f(x) is continuous and differentiable and does not change concavity over [1, 7]. Determine the concavity of f(x) using the following data.

x	1	3	5	7
f(x)	1	5	7	8