Continuity and One-Sided Limits

w-up:

FYI: Intervals of *x*-values will primarily be given in interval notation throughout this course.

EX) $-7 \le x < 8$ will be written as [-7, 8)

EX) $x < -6$ will be written as _____

EX) $x \le 3 \text{ or } x > 5$ will be written as _____

Continuous Function(kindergarten def'n): A function is continuous wherever you do not have to pick up your pencil to draw it's graph.

POLYNOMIALS ARE CONTINOUS EVERYWHERE!

Sketch the following piecewise function

$$y = \begin{cases} x, & x \le 0 \\ 3x+1, & x > 0 \end{cases}$$

At what *x*-value(s) is the graph above discontinuous?

List intervals of continuity for the graph.

How can we change the function above so it is continous for all real numbers?

So, where can discontinuity occur?

Types of Discontinuity

Removable Discontinuity: Can be made continuous by changing just **ONE** point

Non-Removable Discontinuity: Function needs **MORE THAN ONE** point to be made continuous

One Sided Limits

Left Hand Limit: *y*-value approached as *x* approaches "c" from the left

Denoted as $\lim_{x \to c^-}$

Right Hand Limit: *y*-value approached as *x* approaches "c" from the right

Denoted as $\lim_{x \to c^+}$

Reminder: The $\lim_{x \to c}$ only exists when the left and right hand limits are the same!

EX) Find each limit

$$\lim_{x \to 0^+} \frac{|x|}{x}$$

$$\lim_{x \to 0^-} \frac{|x|}{x}$$

$$\lim_{x \to 0} \frac{|x|}{x}$$

Calculus Definition for Continuity

For a function to be continuous at $x = c$, $f(c)$ must be defined and EQUAL to $\lim_{x \to c} f(x)$ (implying the left and right hand limits are equal)

AP EXERCISES

EX) Use the calculus definition of continuity to determine if the following function is continuous. Explain!

$$y = \begin{cases} x^2 + 1, & x \leq 2 \\ 3x - 1, & x > 2 \end{cases}$$

EX) Use your graphing calculator to help you answer the following AP multiple choice question.

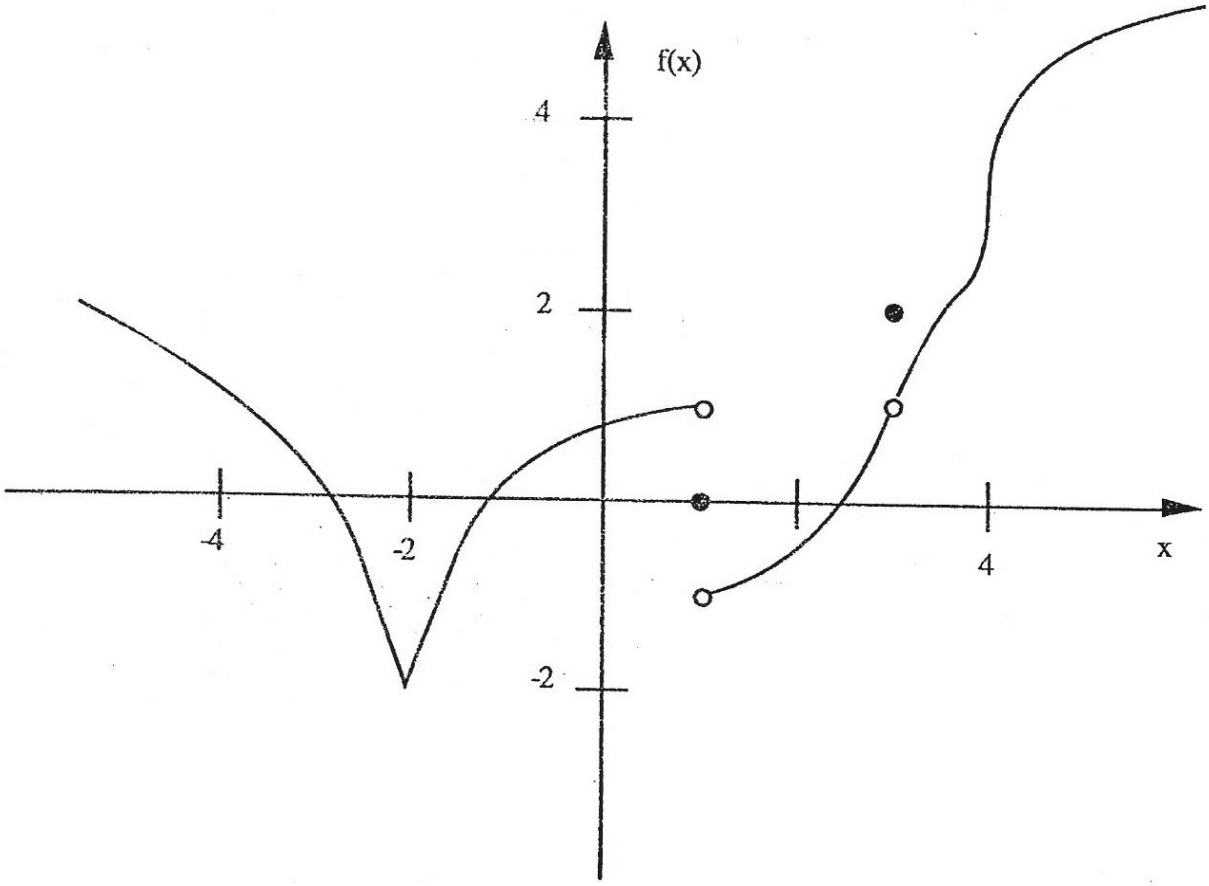
$$f(x) = \begin{cases} k^3 + x & \text{for } x < 3 \\ \dfrac{16}{k^2 - x} & \text{for } x \geq 3 \end{cases}$$

Let f be the function defined above, where k is a positive constant. For what value of k, if any, is f continuous?

(A) 2.081 (B) 2.646 (C) 8.550 (D) There is no such value of k.

Graphical Limit Worksheet

Given the graph of the following function.



Find the following:

A) $f(1)$

B) $\lim_{x\to 1^-} f(x)$

C) $\lim_{x\to 1^+} f(x)$

D) $\lim_{x\to 1} f(x)$

E) $f(3)$

F) $\lim_{x\to 3^-} f(x)$

G) $\lim_{x\to 3^+} f(x)$

H) $\lim_{x\to 3} f(x)$

I) $\lim_{x\to -2} f(x)$

J) $\lim_{x\to 0} f(x)$

K) $\lim_{x\to -3} f(x)$