

Continuity and One-Sided Limits

w-up: Graph the function below and use the graph to help find $\lim_{x \rightarrow 1} f(x)$

$$f(x) = \begin{cases} 3x^2, & x \neq 1 \\ -2, & x = 1 \end{cases}$$

FYI: Intervals of x -values will primarily be given in interval notation throughout this course.

EX) $-7 \leq x < 8$ will be written as $[-7, 8)$

EX) $x < -6$ will be written as _____

EX) $x \leq 3$ or $x > 5$ will be written as _____

Continuous Function(kindergarten def'n): A function is continuous wherever you do **not** have to pick up your pencil to draw it's graph.

POLYNOMIALS ARE CONTINUOUS EVERYWHERE!

Sketch the following piecewise function

$$y = \begin{cases} x, & x \leq 0 \\ 3x + 1, & x > 0 \end{cases}$$

At what x -value(s) is the graph above discontinuous?

List intervals of continuity for the graph.

How can we change the function above so it is continuous for all real numbers?

So, where can discontinuity occur?

Types of Discontinuity

Removable Discontinuity: Can be made continuous by changing just **ONE** point

Non-Removable Discontinuity: Function needs **MORE THAN ONE** point to be made continuous

One Sided Limits

Left Hand Limit: y-value approached as x approaches "c" from the left

Denoted as $\lim_{x \rightarrow c^-}$

Right Hand Limit: y-value approached as x approaches "c" from the right

Denoted as $\lim_{x \rightarrow c^+}$

Reminder: The $\lim_{x \rightarrow c}$ only exists when the left and right hand limits are the same!

EX) Find each limit

$$\lim_{x \rightarrow 0^+} \frac{|x|}{x}$$

$$\lim_{x \rightarrow 0^-} \frac{|x|}{x}$$

$$\lim_{x \rightarrow 0} \frac{|x|}{x}$$

Calculus Definition for Continuity

For a function to be continuous at $x = c$, $f(c)$ must be defined and **EQUAL**

to $\lim_{x \rightarrow c} f(x)$ (implying the left and right hand limits are equal)

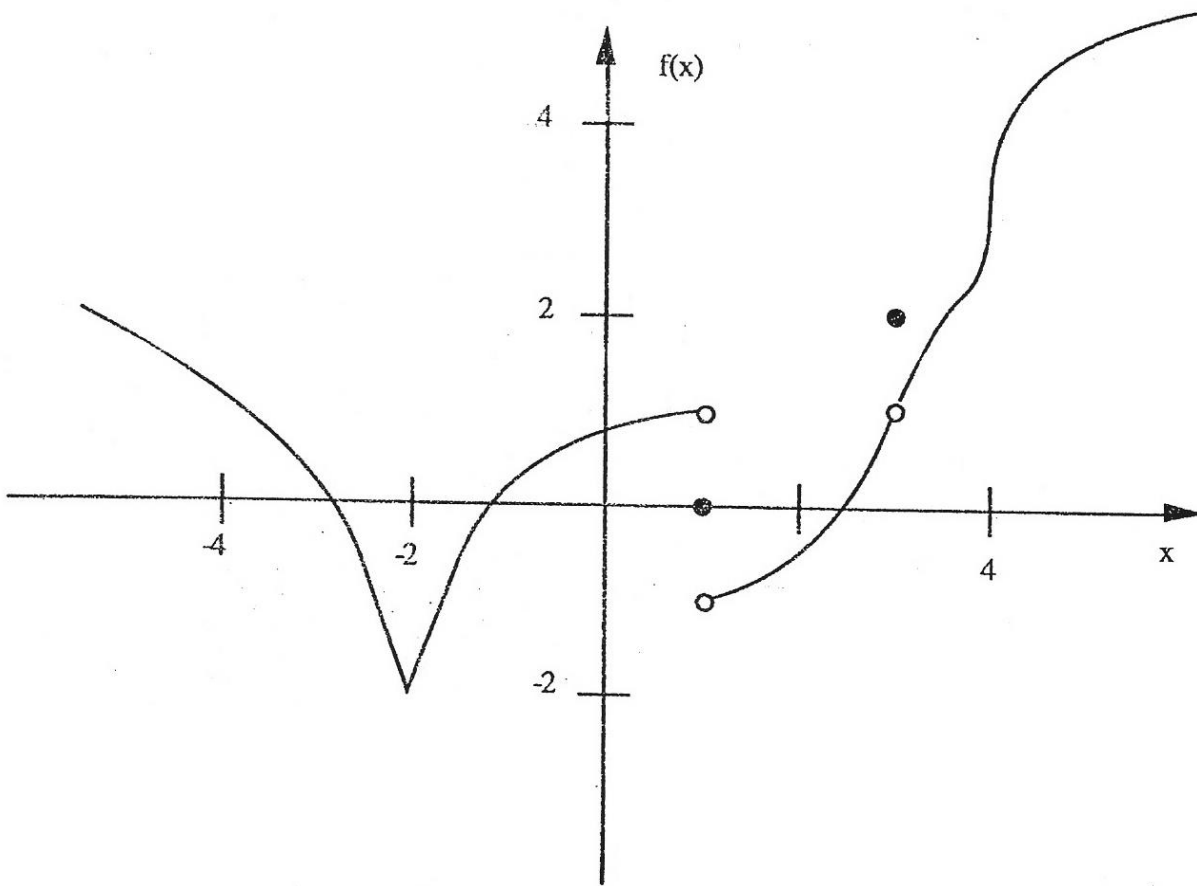
AP EXERCISE

Use the calculus definition of continuity to determine if the following function is continuous. Explain!

$$y = \begin{cases} x^2 + 1, & x \leq 2 \\ 3x - 1, & x > 2 \end{cases}$$

Graphical Limit Worksheet

Given the graph of the following function.



Find the following:

A) $f(1)$

B) $\lim_{x \rightarrow 1^-} f(x)$

C) $\lim_{x \rightarrow 1^+} f(x)$

D) $\lim_{x \rightarrow 1} f(x)$

E) $f(3)$

F) $\lim_{x \rightarrow 3^-} f(x)$

G) $\lim_{x \rightarrow 3^+} f(x)$

H) $\lim_{x \rightarrow 3} f(x)$

I) $\lim_{x \rightarrow -2} f(x)$

J) $\lim_{x \rightarrow 0} f(x)$

K) $\lim_{x \rightarrow -3} f(x)$