Give intervals of differentiability for each function.

1a) $f(x)=|x-3|$ 1b) $f(x)=(x+1)^{2/3}$ 1c) $f(x)=\dfrac{x+4}{x^2-6}$ 1d) $f(x)=\begin{cases} x-5, & x<0 \\ x^3, & x\geq 0 \end{cases}$

$(-\infty,3)\cup(3,\infty)$

Determine with the help of a graphing calculator the intervals of differentiability for each function

2a) $f(x)=\begin{cases} x^2+1, & x\leq 2 \\ 4x-3, & x>2 \end{cases}$

2b) $f(x)=\begin{cases} x, & x\leq 1 \\ x^2, & x>1 \end{cases}$

Continuous since $\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^+} f(x)=5$

Also differentiable at x =2 since

$f'(2)=4$ *for the functions* x^2+1 *and* $4x-3$

So, differentiable over all real numbers!

2c) $f(x)=\begin{cases} (x-1)^3, & x\leq 1 \\ (x-1)^2, & x>1 \end{cases}$

2d) $f(x)=\begin{cases} \frac{1}{2}x+1, & x<2 \\ \sqrt{2x}, & x\geq 2 \end{cases}$

Determine if the following statements are true or false. If false, give a counterexample.

3a) If a function is continuous at a point, then it is differentiable at that point.

3b) If a function has derivatives from both the right and the left at a point, then it is differentiable at that point.

3c) If a function is differentiable at a point, then it is continuous at that point.

4) AP MULTIPLE CHOICE EXAMPLES

1) At $x = 3$, the function given by $f(x) = \begin{cases} x^2 & , \quad x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$ is

(A) undefined.
(B) continuous but not differentiable.
(C) differentiable but not continuous.
(D) neither continuous nor differentiable.
(E) both continuous and differentiable.

2) If $\lim_{x \to 3} f(x) = 7$, which of the following must be true?

I. f is continuous at $x = 3$.
II. f is differentiable at $x = 3$.
III. $f(3) = 7$

(A) None
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

3) If $\lim_{x \to a} f(x) = L$, where L is a real number, which of the following must be true?

(A) $f'(a)$ exists.
(B) $f(x)$ is continuous at $x = a$.
(C) $f(x)$ is defined at $x = a$.
(D) $f(a) = L$
(E) None of the above