AP Calculus Differentiability Critical Homework Answers

1b) $(-\infty,-1)\cup(-1,\infty)$ 1c) $\left(-\infty,-\sqrt{6}\right)\cup\left(-\sqrt{6},\sqrt{6}\right)\cup\left(\sqrt{6},\infty\right)$ 1d) $(-\infty,0)\cup(0,\infty)$

2b) Continuous since $\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^+} f(x)=1$

NOT differentiable at $x = 1$ since

$f'(1)=1$ *for the function* x (from the left)

$f'(1)=2$ *for the function* x^2 (from the right)

So, differentiable over $(-\infty,1)\cup(1,\infty)$

2c) Continuous since $\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^+} f(x)=0$

Also differentiable at $x = 1$ since

$f'(1)=0$ *for the function* $(x-1)^3$ (from the left)

$f'(1)=0$ *for the function* $(x-1)^2$ (from the right)

So, differentiable over all real numbers!

2d) Continuous since $\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^+} f(x)=2$

Also differentiable at $x = 1$ since

$f'(2)=\frac{1}{2}$ *for the function* $\frac{1}{2}x+1$ (from the left)

$f'(2)=\frac{1}{2}$ *for the function* $\sqrt{2x}$ (from the right)

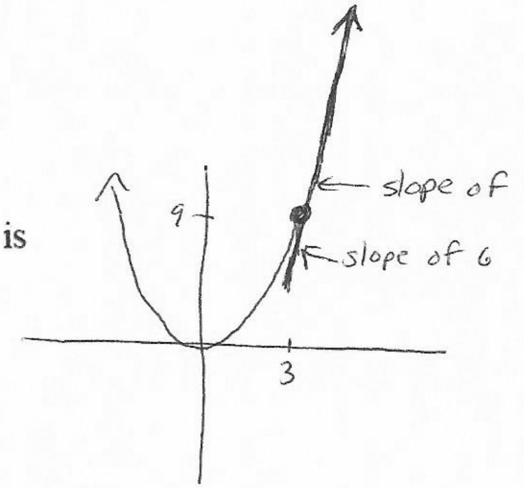
So, differentiable over all real numbers!

3a) FALSE Ex) Absolute value functions are continuous everywhere but NEVER differentiable at the x-value of the vertex.

3b) FALSE Ex) Piecewise functions often have an x-value where the derivative exists from the left and the right, but they are NOT EQUAL. See 2b) from above

3c) TRUE

4) AP MULTIPLE CHOICE EXAMPLES



1) At $x=3$, the function given by $f(x)=\begin{cases} x^2, & x<3 \\ 6x-9, & x\geq 3 \end{cases}$ is

(A) undefined.
(B) continuous but not differentiable.
(C) differentiable but not continuous.
(D) neither continuous nor differentiable.
(E) both continuous and differentiable.

$f'(x) = \begin{cases} 2x, & x<3 \\ 6, & x\geq 3 \end{cases}$

slope from left = slope from right $\Longrightarrow$ at $x=3$

2) If $\lim\limits_{x\to 3} f(x)=7$, which of the following must be true?

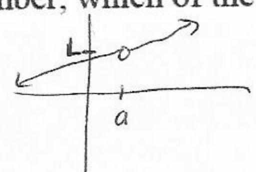
could be a hole in graph at $(3,7)$

I. f is continuous at $x=3$.
II. f is differentiable at $x=3$.
III. $f(3)=7$

(A) None (B) II only (C) III only

(D) I and III only (E) I, II, and III

3) If $\lim\limits_{x\to a} f(x)=L$, where L is a real number, which of the following must be true?



$\leftarrow$ could be this!!

(A) $f'(a)$ exists.
(B) $f(x)$ is continuous at $x=a$.
(C) $f(x)$ is defined at $x=a$.
(D) $f(a)=L$
(E) None of the above

same as #2
but the "all symbolic" version!