Differential Equations and Slope Fields

W-up: Graph *any* solution to $\int x^2\, dx$.

Differential Equation: An equation that represents a derivative (such as $f'(x) = x^2$) often written with the $\frac{dy}{dx}$ notation.

The ***general*** solution to a differential equation is the equation of the antiderivative including the "+C" and represents ALL solutions.

The ***specific*** solution to a differential equation is the equation of the antiderivative with SPECIFIC CONSTANT. The specific constant is that which makes the solution travel through a designated set of point(s).

EX) Find the solution of $\frac{dy}{dx} = x^2$ such that $f(x)$ contains the point $\left(1, \frac{4}{3}\right)$.

EX) Solve the differential $\frac{d^2y}{dx^2} = x^2$ if $f'(0) = 6$ and $f(0) = 3$.

Separation of Variables(often needed on the AP test)

Solving a differential when an implicit derivative is given requires **separation of variables** and integration of both sides of the equation.

EX) Solve the differential $\frac{dy}{dx} = xy^2$ with initial condition $f(2) = 1$.

1) Separate variables. $\frac{dy}{y^2} = x\ dx$

2) Integrate both sides. $\int \frac{1}{y^2} dy = \int x\ dx$

Note: only one "+C" needed since constants on each side of an equation can be combined and do NOT simplify until you solve for "C" $-\frac{1}{y} = \frac{x^2}{2} + C$

3) Use initial condition(the ordered pair (2,1)) and solve for "C" $-\frac{1}{y} = \frac{x^2}{2} - 3$

4) Solve for "y" algebraically(and it won't be fun) $y = \frac{2}{6 - x^2}$

Note: In the event taking the square root of both sides is needed, the "$\pm$" from the right side must be changed to the appropriate one pertaining to the point given in the initial condition!

You can check your answer by finding $y(2)$ and being sure it is "1" **and** finding y' and being sure it is equivalent to xy^2 (also not fun)!

Proof:

$$y = \frac{2}{6-x^2}$$

$y\left(6-x^2\right)=2$ (use product rule and differentiate implicitly)

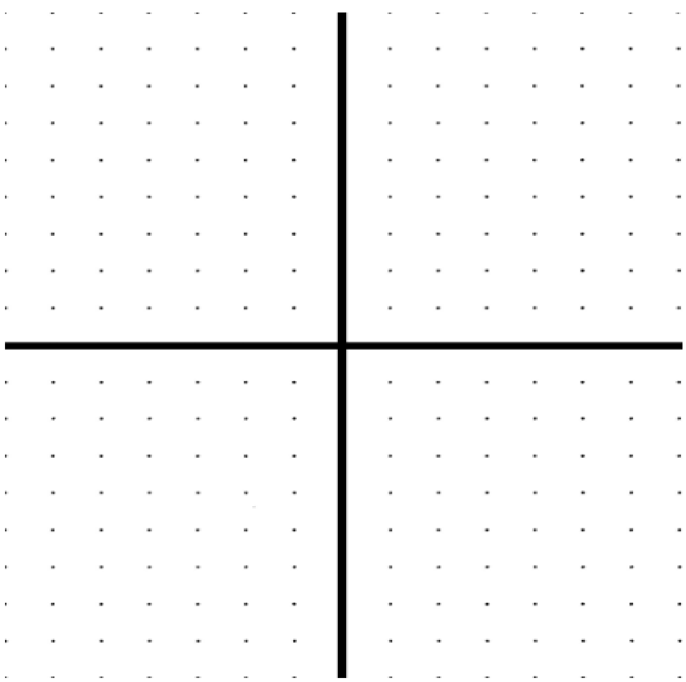
$\frac{dy}{dx}=\frac{2xy}{6-x^2}$ so…. $\frac{dy}{dx}=\frac{2}{6-x^2}\bullet xy$ and finally

$\frac{dy}{dx}=y\bullet xy$ or xy^2

Slope Fields

Slope Field: a graph of the slopes for a general differential equation

EX) Draw a slope field for the differential equation $\frac{dy}{dx}=x^2$

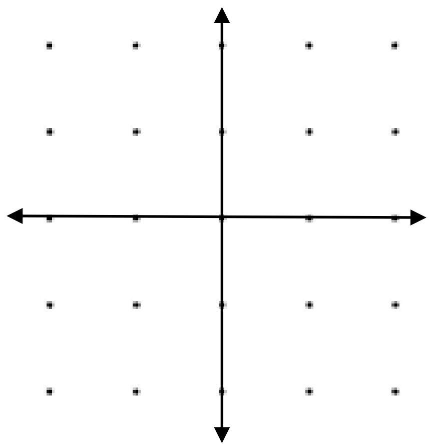


x	$\frac{dy}{dx}$

So, when derivative is *explicit*(containing only x-values) the slope field will be a vertical column of slopes for each x-value

AP EXAMPLE

A) Draw a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{4y}$ for the points shown.



(x, y)	$\frac{dy}{dx}$

B) Find an equation of a tangent line to $y = f(x)$ at the point (2, -2).

Note: " $y =$" equation NOT NEEDED.

C) Find the specific solution $y = f(x)$ of the differential $\frac{dy}{dx} = \frac{x}{4y}$ given the initial condition $f(2) = -2$

D) At what coordinate of *x* is there a horizontal tangent?

E) $y = \sqrt{3}$ is the equation of a horizontal tangent. Is there a min, max or neither at this point?