Solve the differential equation.

1a) $h'(t) = 8t^3 + 5, \; h(1) = -4$

$$h(t) = \int (8t^3 + 5)\,dt = 2t^4 + 5t + C$$

$$h(1) = -4 = 2 + 5 + C \Rightarrow C = -11$$

$$h(t) = 2t^4 + 5t - 11$$

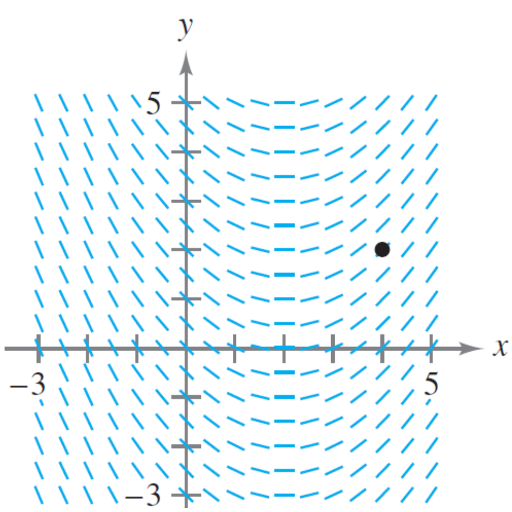
1b) $f'(x) = 6x, \; f(0) = 8$

1c) $f''(x) = 2, \; f'(2) = 5, \; f(2) = 10$

1d) $f''(x) = x^{-3/2}, \; f'(4) = 2, \; f(0) = 0$

Find the specific solution to the differential equation through the given point on the slope field and sketch its graph.

2a) $\dfrac{dy}{dx} = \dfrac{1}{2}x - 1, \; (4, 2)$



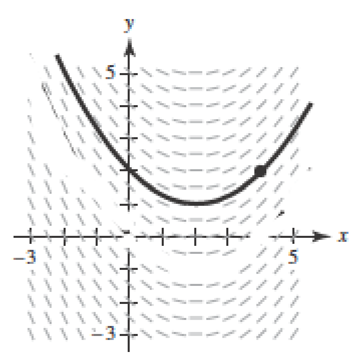
$$\frac{dy}{dx} = \frac{1}{2}x - 1, \; (4, 2)$$

$$y = \frac{x^2}{4} - x + C$$

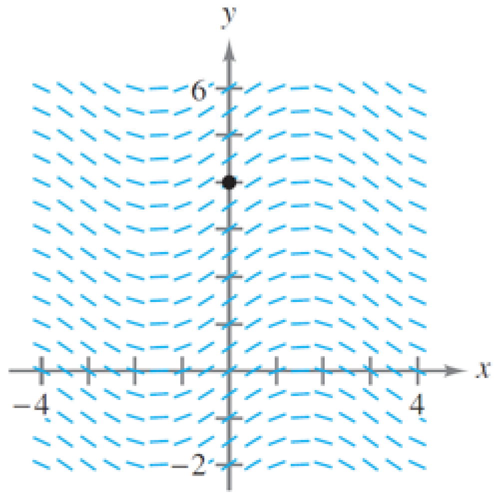
$$2 = \frac{4^2}{4} - 4 + C$$

$$2 = C$$

$$y = \frac{x^2}{4} - x + 2$$



2b) $\dfrac{dy}{dx} = \cos x, \ (0, 4)$



3a) ***Tree Growth*** An evergreen nursery usually sells a certain type of shrub after 6 years of growth and shaping. The growth rate during those 6 years is approximated by $dh/dt = 1.5t + 5$, where t is the time in years and h is the height in centimeters. The seedlings are 12 centimeters tall when planted ($t = 0$).

(a) Find the height after t years.

(b) How tall are the shrubs when they are sold?

(a) $h(t) = \int (1.5t + 5)\, dt = 0.75t^2 + 5t + C$

$h(0) = 0 + 0 + C = 12 \Rightarrow C = 12$

$h(t) = 0.75t^2 + 5t + 12$

(b) $h(6) = 0.75(6)^2 + 5(6) + 12 = 69$ cm

3b) ***Population Growth*** The rate of growth dP/dt of a population of bacteria is proportional to the square root of t, where P is the population size and t is the time in days ($0 \le t \le 10$). That is, $dP/dt = k\sqrt{t}$. The initial size of the population is 500. After 1 day the population has grown to 600. Estimate the population after 7 days.

Consider a particle moving along the *x*-axis where $x(t)$ is the position of the particle at time *t*, $x'(t)$ is its velocity, and $x''(t)$ is its acceleration.

4a) $x(t) = t^3 - 6t^2 + 9t - 2, \quad 0 \le t \le 5$

(a) Find the velocity and acceleration of the particle.

(b) Find the open t-intervals on which the particle is moving to the right.

(c) Find the velocity of the particle when the acceleration is 0.

$x(t) = t^3 - 6t^2 + 9t - 2, \quad 0 \le t \le 5$

(a) $v(t) = x'(t) = 3t^2 - 12t + 9$

$$= 3(t^2 - 4t + 3) = 3(t - 1)(t - 3)$$

$$a(t) = v'(t) = 6t - 12 = 6(t - 2)$$

(b) $v(t) > 0$ when $0 < t < 1$ or $3 < t < 5$.

(c) $a(t) = 6(t - 2) = 0$ when $t = 2$.

$v(2) = 3(1)(-1) = -3$

4b) $x(t) = (t - 1)(t - 3)^2, \quad 0 \le t \le 5$

(a) Find the velocity and acceleration of the particle.

(b) Find the open t-intervals on which the particle is moving to the right.

(c) Find the velocity of the particle when the acceleration is 0.

4c) A particle moves along the x-axis at a velocity of $v(t) = 1/\sqrt{t}$, $t > 0$. At time $t = 1$, its position is $x = 4$. Find the acceleration and position functions for the particle.