Solve the differential equation.

1a) $h'(t) = 8t^3 + 5,\ h(1) = -4$

$$h(t) = \int \left(8t^3 + 5\right) dt = 2t^4 + 5t + C$$

$$h(1) = -4 = 2 + 5 + C \Rightarrow C = -11$$

$$h(t) = 2t^4 + 5t - 11$$

1b) $f'(x) = 6x,\ f(0) = 8$

$f(x) = \int 6x\,dx$

$f(x) = \frac{6x^2}{2} + C$

$f(x) = 3x^2 + C$

$8 = 3(0)^2 + C$

$8 = C$

$\boxed{f(x) = 3x^2 + 8}$

1c) $f''(x) = 2,\ f'(2) = 5,\ f(2) = 10$

$f'(x) = \int 2\,dx$

$f'(x) = 2x + C$

$5 = 2(2) + C$

$5 = 4 + C$

$-4 \quad -4$

$1 = C$

$f'(x) = 2x + 1$

$f(x) = \int 2x + 1\,dx$

$f(x) = \frac{2x^2}{2} + x + C$

$f(x) = x^2 + x + C$

$10 = (2)^2 + (2) + C$

$10 = 4 + 2 + C$

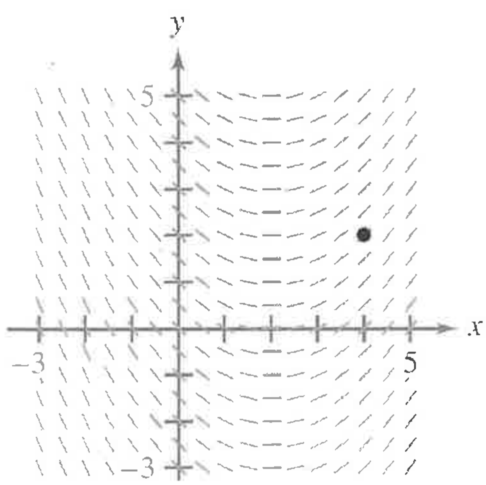
$10 = 6 + C$

$4 = C$

$\boxed{f(x) = x^2 + x + 4}$

Find the specific solution to the differential equation through the given point on the slope field and sketch its graph.

2a) $\dfrac{dy}{dx} = \dfrac{1}{2}x - 1,\ (4, 2)$



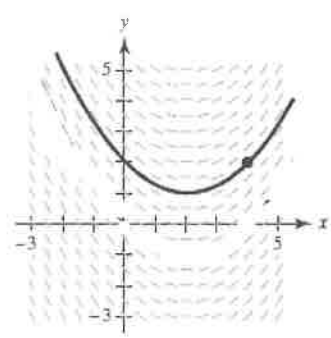
$$\frac{dy}{dx} = \frac{1}{2}x - 1,\ (4, 2)$$

$$y = \frac{x^2}{4} - x + C$$

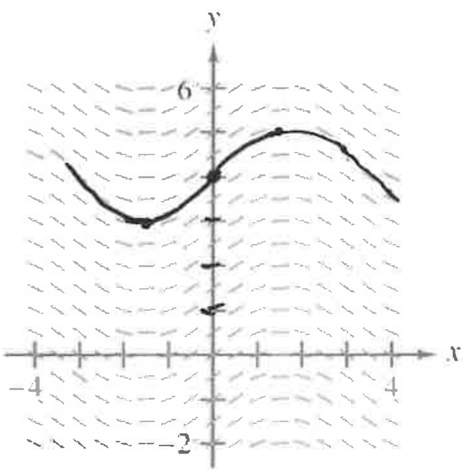
$$2 = \frac{4^2}{4} - 4 + C$$

$$2 = C$$

$$y = \frac{x^2}{4} - x + 2$$



2b) $\frac{dy}{dx} = \cos x,\ (0,\ 4)$



$dy = \cos x\, dx$

$\int 1 dy = \int \cos x\, dx$

$y = \sin x + C$

$4 = \sin(0) + C$

$4 = 0 + C$

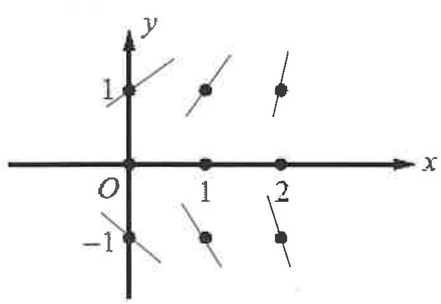
$4 = C$

$\boxed{y = \sin x + 4}$

3a) Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) On the axis provided sketch a slope field for the given differential equation at the nine points indicated.

(x, y)	dy/dx using (x + 1)/y
(0, 1)	1
(0, 0)	Undefined
(0, -1)	-1
(1, 1)	2
(1, 0)	Undefined
(1, -1)	-2
(2, 1)	3
(2, 0)	Undefined
(2, -1)	-3



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $y(1) = \sqrt{3}$.

$$\frac{dy}{dx} = \frac{x+1}{y}$$

$$y\ dy = x + 1\ dx$$

$$\int y\ dy = \int x + 1\ dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + x + C$$

$$\frac{\left(\sqrt{3}\right)^2}{2} = \frac{(1)^2}{2} + 1 + C$$

$$\frac{3}{2} = \frac{1}{2} + 1 + C$$

$$\frac{3}{2} = \frac{3}{2} + C$$

$$0 = C$$

So... $\frac{y^2}{2} = \frac{x^2}{2} + x$

$$y^2 = x^2 + 2x$$

$$y = \pm\sqrt{x^2 + 2x}$$

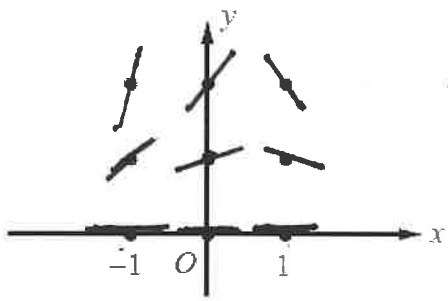
Must be $y = +\sqrt{x^2 + 2x}$ as $\left(1,\ \sqrt{3}\right)$ is on solution

3b)

Consider the differential equation $\frac{dy}{dx} = \frac{y^2(1-2x)}{3}$.

(a) On the axis provided sketch a slope field for the given differential equation at the nine points indicated.

(x,y)	dy/dx
$(-1,0)$	0
$(0,0)$	0
$(1,0)$	0
$(-1,1)$	1
$(0,1)$	$1/3$
$(1,1)$	$-1/3$
$(-1,2)$	4
$(0,2)$	$4/3$
$(1,2)$	$-4/3$



B) $\frac{1}{y^2}\cdot 3\, dy = (1-2x)\,dx$

$\int 3y^{-2}\,dy = \int (1-2x)\,dx$

$\frac{3y^{-1}}{-1} = x - \frac{2x^2}{2} + C$

$-\frac{3}{y} = x - x^2 + C$

$-\frac{3}{(4)} = (\tfrac{1}{2}) - (\tfrac{1}{2})^2 + C \rightarrow \frac{-3}{4} = \frac{1}{4} + C$

$-1 = C$

(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $y(\frac{1}{2}) = 4$.

$\frac{-3}{y} = x - x^2 - 1 \rightarrow \frac{y}{-3} = \frac{1}{x - x^2 - 1}$

$\boxed{y = \frac{-3}{x - x^2 - 1}}$ or $\boxed{y = \frac{3}{x^2 - x + 1}}$

(c) **From previous unit:** Find $\frac{d^2y}{dx^2}$ in terms of x and y.

$\frac{dy}{dx} = \frac{1}{3}y^2\cdot(1-2x) \rightarrow \frac{dy}{dx} = \frac{1}{3}y^2 - \frac{2}{3}xy^2$

$\frac{d^2y}{dx^2} = \frac{2}{3}y\frac{dy}{dx} - \left[\frac{2}{3}x\cdot 2y\frac{dy}{dx} + \frac{2}{3}y^2\right] \rightarrow$ see below

(d) **From previous unit:**

Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $y(\frac{1}{2}) = 4$.

Does f have a relative minimum, a relative maximum, or neither at $x = \frac{1}{2}$? Justify your answer.

Note: we can ask this because $\frac{dy}{dx}\left(\frac{1}{2}, 4\right) = 0$ from above

c) ctd.

$\frac{d^2y}{dx^2} = \frac{2}{3}y\left(\frac{1}{3}y^2 - \frac{2}{3}xy^2\right) - \left[\frac{4}{3}xy\left(\frac{1}{3}y^2 - \frac{2}{3}xy^2\right) + \frac{2}{3}y^2\right]$

$\frac{d^2y}{dx^2} = \frac{2}{9}y^3 - \frac{4}{9}xy^3 - \frac{4}{9}xy^3 + \frac{8}{9}x^2y^3 - \frac{2}{3}y^2$

$\boxed{\frac{d^2y}{dx^2} = \frac{8}{9}x^2y^3 - \frac{8}{9}xy^3 + \frac{2}{9}y^3 - \frac{2}{3}y^2}$

D) Check sign at $(\frac{1}{2}, 4)$ to see if $\frac{d^2y}{dx^2}$ is pos or neg.

$\frac{d^2y}{dx^2}(\frac{1}{2}, 4) = $ " $-$ " $42\frac{2}{3}$

Graph is CC DOWN at this point so f has a relative MAX at $x = 1/2$

4) AP MULTIPLE CHOICE EXAMPLES

1) The solution to the differential equation $\frac{dy}{dx}=\frac{3x^2}{2y}$, where $y(3)=4$, is

MUST BE $y = $ "+" $\sqrt{x^3-11}$ as $(3,4)$ is on the solution

$2y\,dy = 3x^2\,dx$

$\int 2y\,dy = \int 3x^2\,dx$

$\frac{2y^2}{2} = \frac{3x^3}{3} + c$

$y^2 = x^3 + c$

$16 = 27 + c$

$-11 = c$

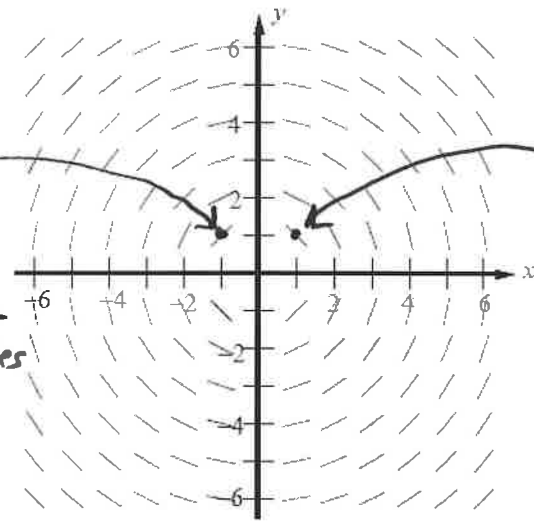
$y^2 = x^3 - 11$

$y = \pm\sqrt{x^3-11}$

(A) $y=\sqrt{\frac{x^3}{3}}+1$ (B) $y=7-\sqrt{\frac{x^3}{3}}$ (C) $y=\sqrt{x^3-9}$ (D) $y=\sqrt{x^3-11}$

2)

$(-1,1)$ slope is a positive number which eliminates "D"



$(1,1)$ slope is a negative number which eliminates "A" & "C"

Shown above is a slope field for which of the following differential equations?

~~(A)~~ $\frac{dy}{dx}=\frac{x}{y}$ $\boxed{\text{(B) } \frac{dy}{dx}=-\frac{x}{y}}$ ~~(C)~~ $\frac{dy}{dx}=\frac{x^2}{y}$ ~~(D)~~ $\frac{dy}{dx}=-\frac{x^2}{y}$

3) If $\frac{dy}{dx}=2y^2$ and if $y=-1$ when $x=1$, then when $x=2$, $y=$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

$\frac{dy}{dx} = 2y^2$

$\frac{1}{y^2}dy = 2\,dx$

$\int \frac{1}{y^2}dy = \int 2\,dx$

$\int y^{-2}dy = \int 2\,dx$

$\frac{y^{-1}}{-1} = 2x + C$

$-\frac{1}{y} = 2x + C$

$-\frac{1}{(-1)} = 2(1) + C$

$1 = 2 + C$

$-1 = C$

$-\frac{1}{y} = 2x - 1$

$\frac{1}{y} = -2x + 1$

$y = \frac{1}{-2x+1}$

$y(2) = \frac{1}{-2(2)+1}$

$= \boxed{\frac{1}{-3}}$

4) The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

(A) $9t^2 + 1$

(B) $3t^2 - 2t + 4$

(C) $t^3 - t^2 + 4t + 6$

(D) $t^3 - t^2 + 9t - 20$

(E) $36t^3 - 4t^2 - 77t + 55$

$V(t) = \int a(t)\,dt$

$V(t) = \int 6t - 2\,dt$

$V(t) = \frac{6t^2}{2} - 2t + C$

$V(t) = 3t^2 - 2t + C$

$25 = 3(3)^2 - 2(3) + C$

$25 = 27 - 6 + C$

$25 = 21 + C$

$4 = C$

$V(t) = 3t^2 - 2t + 4$

$x(t) = \int 3t^2 - 2t + 4\,dt$

$x(t) = 3.\frac{t^3}{3} - \frac{2t^2}{2} + 4t + C$

$x(t) = t^3 - t^2 + 4t + C$

$10 = (1)^3 - (1)^2 + 4(1) + C$

$10 = 4 + C$

$6 = C$

$x(t) = t^3 - t^2 + 4t + 6$