AP Calculus Differentiation of Exponential Functions Critical Homework Answers

1b) $y' = x^3e^x + 3x^2(e^x) = x^2e^x(x+3) = e^x(x^3+3x^2)$

1c) $g'(t) = 3(e^{-t}+e^t)^2(e^t-e^{-t})$

1d) $$y' = \frac{(e^x-1)e^x-(e^x+1)e^x}{(e^x-1)^2} = \frac{-2e^x}{(e^x-1)^2}$$

2b) $$\begin{aligned} g'(t) &= t^2(\ln 2)2^t + (2t)2^t \\ &= t2^t(t\ln 2 + 2) \\ &= 2^t t(2 + t\ln 2) \end{aligned}$$

2c) $$\begin{aligned} h'(\theta) &= 2^{-\theta}(-\pi\sin\pi\theta) - (\ln 2)2^{-\theta}\cos\pi\theta \\ &= -2^{-\theta}\left[(\ln 2)\cos\pi\theta + \pi\sin\pi\theta\right] \end{aligned}$$

3b) $y' = x^2e^x + 2xe^x - 2xe^x - 2e^x + 2e^x = x^2e^x$

$y'(1) = e$

Tangent line: $y - e = e(x-1)$

$y = ex$

4b) $$e^{xy}\left(x\frac{dy}{dx}+y\right)+2x-2y\frac{dy}{dx}=0$$

$$xe^{xy}\frac{dy}{dx}+ye^{xy}+2x-2y\frac{dy}{dx}=0$$

$$xe^{xy}\frac{dy}{dx}-2y\frac{dy}{dx}=-ye^{xy}-2x$$

$$\frac{dy}{dx}\left(xe^{xy}-2y\right)=-ye^{xy}-2x$$

$$\frac{dy}{dx}=\frac{-ye^{xy}-2x}{xe^{xy}-2y}$$

5a) $$\frac{dV}{dt} = -9429e^{-0.6286t}$$

When $t = 5$, $\frac{dV}{dt} \approx -406.89$

5b) $$\frac{dV}{dt} = 20{,}000\left(\ln\frac{3}{4}\right)\left(\frac{3}{4}\right)^t$$

$$\frac{dV}{dt}(4) = 20{,}000\left(\ln\frac{3}{4}\right)\left(\frac{3}{4}\right)^4$$

$$= \$1820.40/\text{year}$$

The car value is decreasing \$1820.40/year in the 4th year!