Extrema on an Interval

How do you prove to someone that a value is a maximum or minimum?

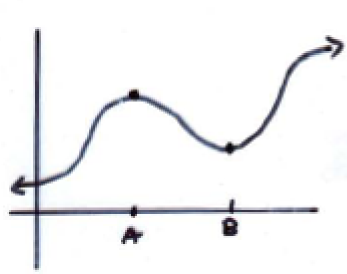
Extrema: $f(c)$ is the MINIMUM of f on [a,b] if $f(c) \leq f(x)$ for ALL x on [a,b] while $f(c)$ is the MAXIMUM of f on [a,b] if $f(c) \geq f(x)$ for ALL x on [a,b]

Extreme Value Theorem

If f is continuous on a closed interval [a,b], then f has both a minimum and a maximum on the interval.

Maximum and Minimum MUST Exist!

Relative Extrema: Maximum or Minimum that occur over an open interval

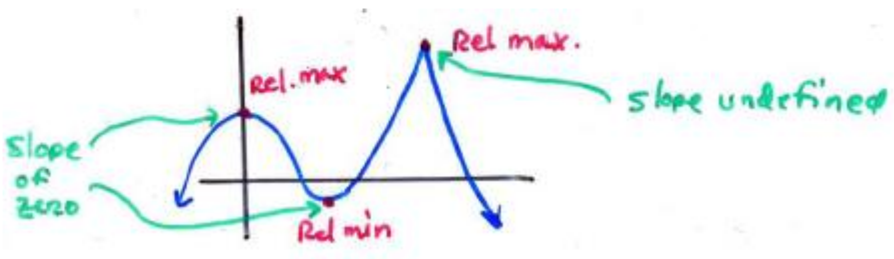


Relative maximum occurs at x = A and a
Relative minimum occurs at x = B

Note: When relative extrema are the extrema of the ENTIRE FUNCTION they would be called ABSOLUTE EXTREMA.

Where can relative extrema occur(a calculus reason)?

Relative extrema occur at a smooth turn(slope of tangent line is ZERO) or a sharp turn(slope of the tangent line is UNDEFINED).



Critical Number: an x-value of a function where the slope of the tangent line is either zero or undefined.

Using the Extreme Value Theorem for finding extrema on a closed interval [a,b]

Note that the function must be continuous on the interval [a,b]!

1) Evaluate $f(a)$ and $f(b)$

2) Find the critical numbers using the derivative. Evaluate those values in the function as well.

The largest of 1 and 2 is the MAXIMUM and the smallest is the MINIMUM.

EX) Find all extrema for each function over the given interval.

A) $f(x)=x^3-3x^2$ over [-2,2] B) $f(x)=\dfrac{x}{x-2}$ over [3,5] C) $f(x)=2x-3x^{2/3}$ over [-1,8]

Note: If the Extreme Value Theorem CANNOT be applied because an "open" interval is used or because of discontinuity on the interval, a maximum and minimum are not both guaranteed but still possible.

EX) Locate extrema of each function using the given interval.

A) $f(x) = 2x + 3$ over

- A) [0,2]
- B) (0,2)
- C) [0,2)
- D) (0,2]

B) $f(x) = x^2 + 2$ over

- A) [-1,2]
- B) (-1,2)
- C) [-1,2)
- D) (-1,2]