The 1st and 2nd Fundamental Theorem of Calculus

w-up: AP multiple choice # 22(non-calculator) and #77(calculator)

The Fundamental Theorem of Calculus(FTC)

If f is continuous on the closed interval $[a, b]$ then

$\int_a^b f(x)dx = F(b) - F(a)$ where F(x) is the antiderivative

Note: When evaluating a definite integral using the FTC the constant of integration "+C" is NOT needed because it will always cancel out due to the subtraction between $F(a)$ and $F(b)$.

EX) Evaluate each Integral using the FTC

A) $\int_0^4 \frac{1}{2}x^2 + 1\, dx$

B) $\int_1^4 t^3 - 9t\, dt$

C) $\int_1^8 \frac{x^2}{2\sqrt[3]{x}}\, dx$

The 2nd Fundamental Theorem of Calculus

If $G(x) = \int x^2 + 1\, dx$, find $G'(x)$.

So…the derivative of a given antiderivative returns the function used for the integral. This is the basis of the 2nd Fundamental Theorem of Calculus!

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

Where a represents a constant

Prove the 2nd FTC by **evaluating the integral** and **differentiating the result** for the following examples.

EX) $\frac{d}{dx}\left[\int_1^x t^3 + t\, dt\right]$

Note: the constant used has no bearing on the final answer since it only effects the value of the constant which goes away once differentiated.

EX) $\frac{d}{dx}\left[\int_2^{\sin x} t^3\, dt\right]$

Note: When the upper limit is NOT just "*x*" the answer must contain(be multiplied by) "the derivative of that expression" because of the chain rule used when differentiating(and that expression represents our "stuff").

Use the 2nd FTC to evaluate each expression

EX) $\frac{d}{dx}\left[\int_1^x \sqrt{3t-4}\ dt\right]$ EX) $\frac{d}{dx}\left[\int_1^{\sec x} \frac{t+1}{t-1}\ dt\right]$ EX) $\frac{d}{dx}\left[\int_x^{2x} \tan t - 3\ dt\right]$ EX) $\frac{d}{dx}\left[\int_{x^2}^{5} \sin t\ dt\right]$

Hint: use properties first!

Note: Always write the constant as the lower limit of integration(or use properties to make sure it is).

AP Application using the 2nd FTC(graphing calculator allowed)

Given $f(x) = \int_5^x t^2 - t - 12\ dt$

A) Find the intervals of increase/decrease for $f(x)$ and identify the ordered pairs of any min/max/plateau.

B) Find the intervals of concavity for $f(x)$ and identify the ordered pairs of any inflection points.

C) Sketch the graph of $f(x)$.