Implicit Differentiation

w-up: AP Multiple Choice #1, 14(non-calculator) and #76(calculator)

Graph $x^2+y^2=25$ and estimate the slope of the tangent line at x = 0, and $x=\pm 5$.

When finding the derivative of NON-FUNCTIONS, it is not necessary to solve for y. In fact, sometimes it is impossible to solve for y though we can still find an expression for the derivative.

Find $\frac{dy}{dx}$ if $y=x\bullet\sin^2 x$

We may only differentiate with respect to **one variable**(usually it has been "x"). So, when differentiating a variable other than x requires us to differentiate as a composite function using $\frac{dy}{dx}$ as the symbolic form for the derivative of the inner function.

Find $\frac{dy}{dx}$ if $y=x\bullet y^2$ using the example above as a model.

$$\frac{dy}{dx} = x\bullet 2y\frac{dy}{dx}+y^2\bullet 1$$

Needed because I do not know what y is in terms of x so this is how we say the derivative of the inner function "in terms of x"

Now solve for $\frac{dy}{dx}$

$$\frac{dy}{dx}-x\bullet 2y\frac{dy}{dx}=y^2$$

$$\frac{dy}{dx}(1-x\bullet 2y)=y^2$$

$$\frac{dy}{dx}=\frac{y^2}{(1-2xy)}$$

Note: This derivative needs both an x and y coordinate to find a slope(for the tangent line) of the graph anywhere.

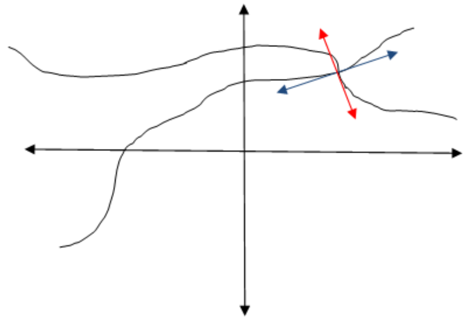
EX) Find the slope of the graph of $x^2+y^2=25$ at x = 3 first by solving for y and using the chain rule and secondly by using implicit differentiation.

EX) Find $\frac{dy}{dx}$ for the equation $y^2 = 4x-3x^2y-6y$

Normal Line: Line **perpendicular** to the tangent line.

EX) Find the *equation* of the normal line to the graph of $x^2+y^2=25$ at x = 3.

Orthogonal graphs are graphs that have perpendicular tangent lines at their point of intersection. So, algebraically we can tell if graphs are orthogonal if their slopes are opposite inverses at their point of intersection.



Notation for the 2nd Derivative

$\frac{d^2y}{dx^2}$ means the Second Derivative of y with respect to x

$$y = 3x^4$$

$$\frac{dy}{dx} = 12x^3$$

$$\frac{d^2y}{dx^2} = 36x^2 \quad \text{(in terms of } x\text{)}$$

or

$$\frac{d^2y}{dx^2} = 36\sqrt{y/3} \quad \text{(in terms of } y\text{)}$$

Original equation solved for "x^2" and substituted in its place.