

Find dy/dx of each equation.

1a) $x^3 - xy + y^2 = 7$

1b) $x^{1/2} + y^{1/2} = 16$

$$3x^2 - xy' - y + 2yy' = 0$$

$$(2y - x)y' = y - 3x^2$$

$$y' = \frac{y - 3x^2}{2y - x}$$

1c) $x^3y^3 - y = x$

1d) $\sin x + 2 \cos 2y = 1$

1e) $x^3 - 3x^2y + 2xy^2 = 12$

Evaluate the derivative of the function at the given point.

2a) $y^2 = \frac{x^2 - 49}{x^2 + 49}, (7, 0)$

2b) $\tan(x + y) = x, (0, 0)$

Find the point(s) at which the graph of the equation has a vertical *or* horizontal tangent line.

3a) $25x^2 + 16y^2 + 200x - 160y + 400 = 0$

Find $\frac{d^2y}{dx^2}$ in terms of x and y .

4a) $x^2 - y^2 = 36$

4b) $y^2 = x^3$

$$\begin{aligned}
 x^2 - y^2 &= 36 & \frac{dy}{dx} &= \frac{x}{y} \\
 2x - 2y \frac{dy}{dx} &= 0 & \frac{d^2y}{dx^2} &= \frac{y \cdot 1 - x \cdot 1}{y^2} \frac{dy}{dx} \\
 -2y \frac{dy}{dx} &= -2x & \frac{d^2y}{dx^2} &= \frac{y - x \left(\frac{x}{y} \right)}{y^2} \\
 \frac{dy}{dx} &= \frac{-2x}{-2y} & \frac{d^2y}{dx^2} &= \frac{y - \frac{x^2}{y}}{y^2} \cdot \frac{y}{y} \\
 \frac{dy}{dx} &= \frac{x}{y} & \frac{d^2y}{dx^2} &= \frac{y^2 - x^2}{y^3}
 \end{aligned}$$

Write the equation of the NORMAL line(s) to each equation at the given x -value.

5a) $y^2 = 2x + 14$ at $x = 1$

When $x = 1$ $y = \pm 4$

Derivative:

$$2y \frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = \frac{1}{y}$$

$$\frac{dy}{dx}(1, 4) = \frac{1}{4} \quad \text{and} \quad \frac{dy}{dx}(1, -4) = -\frac{1}{4}$$

Normal line slopes: -4 at $(1, 4)$ and 4 at $(1, -4)$

So, equations of NORMAL lines are:

$$y = 4(x - 1) - 4 \quad \& \quad y = -4(x - 1) + 4$$

5b) $3y^2 - 2x^2 - 4 = 0$ at $x = 1$

6) AP MULTIPLE CHOICE EXAMPLES

1) If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y , $\frac{dy}{dx} =$

(A) $-\frac{x^2 + y}{x + 2y^2}$ (B) $-\frac{x^2 + y}{x + y^2}$ (C) $-\frac{x^2 + y}{x + 2y}$

(D) $-\frac{x^2 + y}{2y^2}$ (E) $\frac{-x^2}{1 + 2y^2}$

2) If $x + 2xy - y^2 = 2$, then at the point $(1, 1)$, $\frac{dy}{dx}$ is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{3}{2}$ (E) nonexistent

3) If $\tan(xy) = x$, then $\frac{dy}{dx} =$

(A) $\frac{1 - y \tan(xy) \sec(xy)}{x \tan(xy) \sec(xy)}$ (B) $\frac{\sec^2(xy) - y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy) - y}{x}$