Find dy/dx of each equation.

1a) $x^3 - xy + y^2 = 7$

$$3x^2 - xy' - y + 2yy' = 0$$
$$(2y - x)y' = y - 3x^2$$
$$y' = \frac{y - 3x^2}{2y - x}$$

1b) $x^{1/2} + y^{1/2} = 16$

$$\tfrac{1}{2}x^{-1/2} + \tfrac{1}{2}y^{-1/2}\frac{dy}{dx} = 0$$
$$x^{-1/2} + y^{-1/2}\frac{dy}{dx} = 0$$
$$y^{-1/2}\frac{dy}{dx} = -x^{-1/2}$$
$$\frac{dy}{dx} = \frac{-x^{-1/2}}{y^{-1/2}} = \boxed{-\frac{\sqrt{y}}{\sqrt{x}}}$$

1c) $x^3y^3 - y = x$

$$x^3 \cdot 3y^2\frac{dy}{dx} + y^3 \cdot 3x^2 - \frac{dy}{dx} = 1$$
$$x^3 \cdot 3y^2\frac{dy}{dx} - \frac{dy}{dx} = 1 - 3x^2y^3$$
$$\frac{dy}{dx}(3x^3y^2 - 1) = 1 - 3x^2y^3$$
$$\frac{dy}{dx} = \boxed{\frac{1 - 3x^2y^3}{3x^3y^2 - 1}}$$

1d) $\sin x + 2\cos 2y = 1$

$$\cos x + 2(-\sin 2y) \cdot 2\frac{dy}{dx} = 0$$
$$-4\sin 2y\frac{dy}{dx} = -\cos x$$
$$\frac{dy}{dx} = \boxed{\frac{\cos x}{4\sin 2y}}$$

1e) $x^3 - 3x^2y + 2xy^2 = 12$

$$3x^2 - \left[3x^2\frac{dy}{dx} + 6xy\right] + 2x \cdot 2y\frac{dy}{dx} + 2y^2 = 0$$
$$3x^2 - 3x^2\frac{dy}{dx} - 6xy + 4xy\frac{dy}{dx} + 2y^2 = 0$$
$$-3x^2\frac{dy}{dx} + 4xy\frac{dy}{dx} = -3x^2 + 6xy - 2y^2$$
$$\frac{dy}{dx}(-3x^2 + 4xy) = -3x^2 + 6xy - 2y^2$$
$$\frac{dy}{dx} = \boxed{\frac{-3x^2 + 6xy - 2y^2}{-3x^2 + 4xy}}$$

Evaluate the derivative of the function at the given point.

2a) $y^2 = \dfrac{x^2 - 49}{x^2 + 49}$, $(7, 0)$

$$2y\frac{dy}{dx} = \frac{(x^2+49) \cdot 2x - 2x(x^2-49)}{(x^2+49)^2}$$
$$\frac{dy}{dx} = \frac{\cancel{2x^3} + 98x - \cancel{2x^3} + 98x}{2y(x^2+49)^2}$$
$$\frac{dy}{dx}(7,0) = \frac{196(7)}{2 \cdot 0 \cdot (98)^2}$$

UNDEFINED

2b) $\tan(x + y) = x$, $(0, 0)$

$$\sec^2(x+y) \cdot \left(1 + \frac{dy}{dx}\right) = 1$$
$$1 + \frac{dy}{dx} = \frac{1}{\sec^2(x+y)}$$
$$\frac{dy}{dx} = \frac{1}{\sec^2(x+y)} - 1$$
$$\frac{dy}{dx} = \cos^2(x+y) - 1$$
$$\frac{dy}{dx}(0,0) = (\cos(0+0))^2 - 1$$
$$= (\cos(0))^2 - 1$$
$$= 1^2 - 1 = \boxed{0}$$

$25x^2+16(5)^2+200x-160(5)+400=0$

$25x^2+200x=0$

$25x\cdot(x+8)=0$ so.. $x=0$ or -8

$25(-4)^2+16y^2+200(-4)-160y+400=0$

$=16y^2-160y=0$

$=16y(y-10)=0$

$y=0$ or $+10$

Find the point(s) at which the graph of the equation has a vertical *or* horizontal tangent line.

slope undef
(0, 5)
(-8, 5)

slope of zero
(-4, 0)
(-4, 10)

3a) $25x^2 + 16y^2 + 200x - 160y + 400 = 0$

$50x + 32y\frac{dy}{dx} + 200 - 160\frac{dy}{dx} + 0 = 0$

$32y\frac{dy}{dx} - 160\frac{dy}{dx} = -50x - 200$

$\frac{dy}{dx}(32y-160) = -50x-200$

$\frac{dy}{dx} = \frac{-50x-200}{32y-160}$

If $\frac{dy}{dx} = \frac{-50x-200}{32y-160}$

slope undef when
$32y-160=0$
$y=\frac{160}{32}$
$y=5$

slope zero when
$-50x-200=0$
$x=\frac{200}{-50}$
$x=-4$

Find $\frac{d^2y}{dx^2}$ in terms of *x* and *y*.

4a) $x^2 - y^2 = 36$

$$x^2 - y^2 = 36$$
$$2x - 2yy' = 0$$
$$y' = \frac{x}{y}$$
$$x - yy' = 0$$
$$1 - yy'' - (y')^2 = 0$$
$$1 - yy'' - \left(\frac{x}{y}\right)^2 = 0$$
$$y^2 - y^3y'' = x^2$$
$$y'' = \frac{y^2 - x^2}{y^3} = -\frac{36}{y^3}$$

4b) $y^2 = x^3$

$2y\frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{2y}$

$\frac{d^2y}{dx^2} = \frac{2y\cdot 6x - 3x^2\cdot 2\frac{dy}{dx}}{(2y)^2}$

$\frac{d^2y}{dx^2} = \frac{12xy - \cancel{6}^{3}x^2\cdot\left(\frac{3x^2}{\cancel{2}y}\right)}{4y^2}$

$\frac{d^2y}{dx^2} = \frac{12xy - \frac{9x^4}{y}}{4y^2} \cdot \frac{y}{y}$

$= \boxed{\frac{12xy^2-9x^4}{4y^3}}$

Write the equation of the NORMAL line(s) to each equation at the given *x*-value.

5a) $y^2 = 2x + 14$ at $x = 1$

When $x = 1$ $y = \pm 4$

Derivative:

$2y\frac{dy}{dx} = 2$

$\frac{dy}{dx} = \frac{1}{y}$

$\frac{dy}{dx}(1, 4) = \frac{1}{4}$ and $\frac{dy}{dx}(1, -4) = -\frac{1}{4}$

Normal line slopes: -4 at $(1, 4)$ and 4 at $(1, -4)$

So, equations of NORMAL lines are:

$y = 4(x-1) - 4$ & $y = -4(x-1) + 4$

5b) $3y^2 - 2x^2 - 4 = 0$ at $x = 1$

when $x = 1$ $\Rightarrow$ $3y^2 - 2(1)^2 - 4 = 0$

$3y^2 - 6 = 0$

$y^2 = 2$

$y = \pm\sqrt{2}$

$6y\frac{dy}{dx} - 4x - 0 = 0$

$\frac{dy}{dx} = \frac{4x}{6y}$

$\frac{dy}{dx} = \frac{2x}{3y}$ $\quad$ $\frac{dy}{dx}(1, \sqrt{2}) = \frac{2}{3\sqrt{2}}$

So... $y - \sqrt{2} = \frac{-3\sqrt{2}}{2}(x-1)$

$\frac{dy}{dx}(1, -\sqrt{2}) = \frac{2}{-3\sqrt{2}}$

So.. $y + \sqrt{2} = \frac{3\sqrt{2}}{2}(x-1)$

1) If $x^3+3xy+2y^3=17$, then in terms of x and y, $\frac{dy}{dx}=$

(A) $-\frac{x^2+y}{x+2y^2}$ (B) $-\frac{x^2+y}{x+y^2}$ (C) $-\frac{x^2+y}{x+2y}$

(D) $-\frac{x^2+y}{2y^2}$ (E) $\frac{-x^2}{1+2y^2}$

$3x^2+3x\frac{dy}{dx}+3y+6y^2\frac{dy}{dx}=0$

$3x\frac{dy}{dx}+6y^2\frac{dy}{dx}=-3x^2-3y$

$\frac{dy}{dx}(3x+6y^2)=-3x^2-3y$

$\frac{dy}{dx}=\frac{-3x^2-3y}{3x+6y^2}$

$=\frac{-\cancel{3}(x^2+y)}{\cancel{3}(x+2y^2)}$

$=-\frac{(x^2+y)}{x+2y^2}$

2) If $x+2xy-y^2=2$, then at the point $(1,1)$, $\frac{dy}{dx}$ is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{3}{2}$ (E) nonexistent

$1+2x\frac{dy}{dx}+2y-2y\frac{dy}{dx}=0$

$2x\frac{dy}{dx}-2y\frac{dy}{dx}=-2y-1$

$\frac{dy}{dx}(2x-2y)=-2y-1$

$\frac{dy}{dx}=\frac{-2y-1}{2x-2y}$ $\quad \frac{dy}{dx}(1,1)=\frac{-2-1}{2-2}=\frac{-3}{0}$ undef.

3) If $\tan(xy)=x$, then $\frac{dy}{dx}=$

(A) $\frac{1-y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy)-y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy)-y}{x}$

$\sec^2(xy)\cdot\left[x\frac{dy}{dx}+y\right]=1$

$x\frac{dy}{dx}+y=\frac{1}{\sec^2(xy)}$

$x\frac{dy}{dx}+y=\cos^2(xy)$

$x\frac{dy}{dx}=\cos^2(xy)-y$

$\frac{dy}{dx}=\frac{\cos^2(xy)-y}{x}$