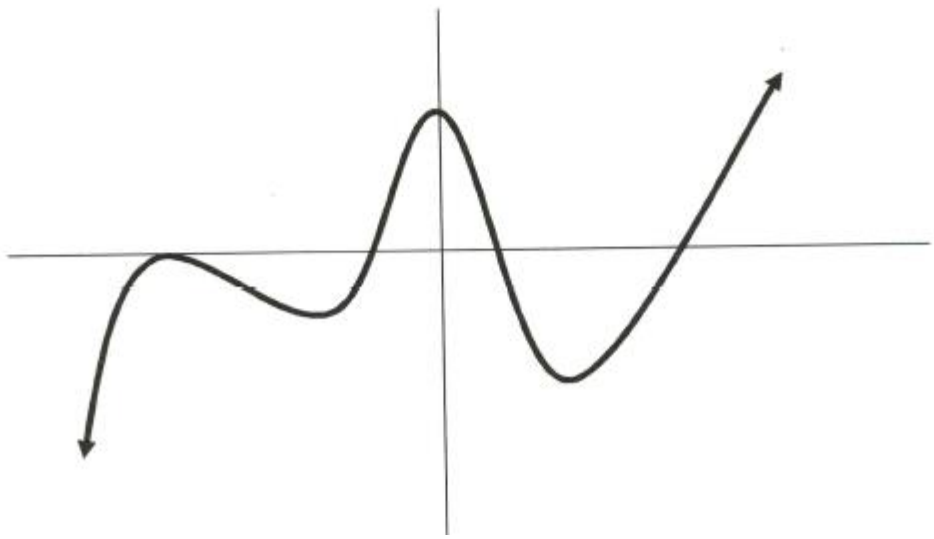
Increasing and Decreasing Functions & The First Derivative Test

w-up: AP multiple choice #80(calculator allowed)

Draw 15 different tangent lines to the graph below:



What is true about the slopes of a graph when the function increases(y-values increase as x-values increase left to right)?

What is true about the slopes of a graph when the function decreases(y-values decrease as x-values increase left to right)?

Tangent line slopes are POSITIVE on intervals where a graph increases and NEGATIVE on intervals where a graph decreases.

Since a graph can only change from increasing to decreasing(or vice versa) at a critical point, Calculus can be used for find intervals of increase/decrease and ordered pairs for maximums, minimums and plateaus.

THEOREM 3.5 Test for Increasing and Decreasing Functions

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.

Using the First Derivative Test to find intervals of increase/decrease and x -values for relative maximums/minimums and plateaus.

1) Find critical numbers and place them on a number line.

2) Pick any value in between these critical values and evaluate them in the derivative to determine if the value is on an increasing(positive value) or decreasing(negative value) interval and mark with a +/- above the number line. List CLOSED intervals of increase and decrease.

3) If there is a sign change over a critical point, then POSSIBLY a min./max. occurs there. If no sign change occurs over a critical point then POSSIBLY a plateau occurs. Remember, **neither** of these could be true if the critical point represents the x-value of a vertical *asymptote*.

EX)Give an example of function with critical value of x =1 which changes from increasing to decreasing at this value but **IS NOT** the x-value of a relative maximum.

Maximums/Minimums/Plateaus

When a graph changes from **increasing to decreasing** over a DEFINED critical point then a MAXIMUM occurs at that x-value!

When a graph changes from **decreasing to increasing** over a DEFINED critical point then a MINIMUM occurs at that x-value!

When a graph DOES NOT change from **dec. to inc. or vice-versa** over a DEFINED critical point then a PLATEAU occurs at that x-value!

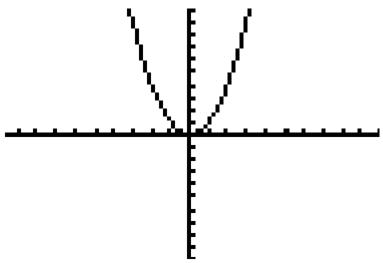
EX For each function, determine(without a calculator) intervals of inc/dec and identify any ordered pairs of relative maximums/minumums or plateaus.

A) $f(x)=x^3-\frac{3}{2}x^2$ B) $f(x)=2x^3-3x^2-36x+14$ C) $f(x)=\left(x^2-9\right)^{2/3}$

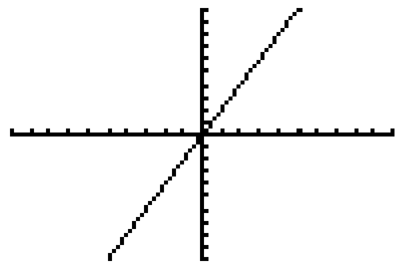
READING A DERIVATIVE GRAPH TO DETERMINE TRAITS OF ORIGINAL FUNCTION

Compare the graphs of a function and its derivative below.

$f(x) = x^2$



$f'(x) = 2x$



Notice when reading the graph of the derivative, the *y*-values represent the slope of the graph of the original function at the same *x*-value.

When reading a derivative graph($f'(x)$):

***x*-intercepts represent *x*-values where horizontal tangents occur on original function**

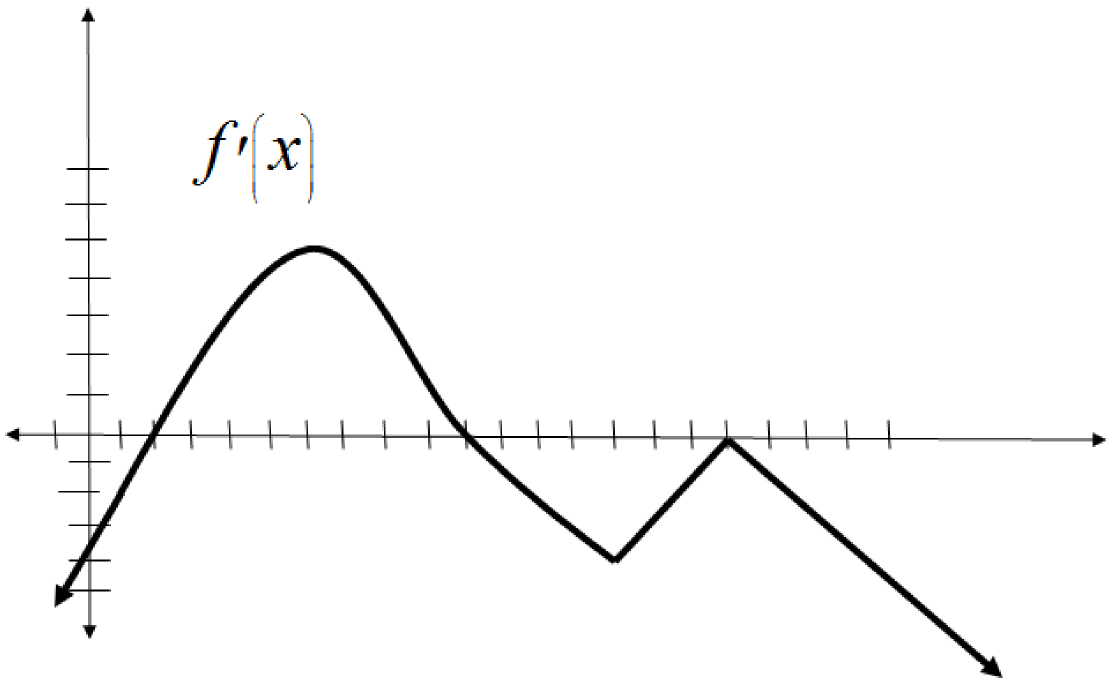
AND

intervals where there are positive *y*-values(above the *x*-axis) *on the derivative* represent intervals of increase on the original function

AND

intervals where there are negative *y*-values(below the *x*-axis) *on the derivative* represent intervals of decrease on the original function

EX) Use the graph of the **derivative** below to list intervals of increase/decrease **on $f(x)$** and identify any x-values of minimums, maximums and plateaus.



Using the Calculator to Determine Intervals of INC/DEC

If $f'(x) = \sin x \bullet \ln(x^2 - 1)$, determine the interval(s) of increase/decrease of $f(x)$ over [2, 10]. Also, determine the x-values of any relative extrema on $f(x)$.