

Indeterminate Forms and L'Hopital's Rule

W-up: Use your graphing calculator to evaluate the following limit graphically

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$$

L'Hopital's Rule : Method of using differentiation to find limits that cannot be solved algebraically. Use direct substitution to try and evaluate the limit. If the indeterminate forms of $\frac{0}{0}$ OR $\frac{\pm\infty}{\pm\infty}$ then the limit of the result of differentiating the numerator and denominator *individually* will be equivalent to the original expression. **Note**: We are NOT using the quotient rule to differentiate the rational expression.

L'Hopital's Rule

$$\text{If } \lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\pm\infty}{\pm\infty} \text{ then } \lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$$

provided the limits exist!

EX) Use L'Hopital's Rule to evaluate $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$ to verify result from above.

EX) Use L'Hopital's Rule to evaluate each limit. Note: the rule may have to be applied more than one time to evaluate the limit.

A) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$

B) $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$

C) $\lim_{x \rightarrow -\infty} \frac{x^2}{e^{-x}}$

Note: L'Hopital's Rule is true ONLY for the indeterminate forms of $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$.

Any other indeterminate forms $\left(0 \bullet \infty, 1^\infty, 0^0, \infty - \infty\right)$ must be rewritten so that

$\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$ are obtained.