

## Infinite Limits

w-up: Make a basic sketch for each graph

$$A) f(x) = \frac{1}{x} \quad B) f(x) = \frac{1}{x^2} \quad C) f(x) = \frac{2}{x-4} \quad D) f(x) = \frac{2}{(x-4)^2}$$

What is the impact on each graph if the rational expression is NEGATIVE?

**Infinite Limit:** A limit that does not exist but may **increase** without bound ( $+\infty$ ) or **decrease** without bound ( $-\infty$ )

Note: the infinity symbol ( $\pm\infty$ ) tells us more specifically how the limit fails to exist.

Where do infinite limits occur on a graph?

$$\text{EX1) } A) \lim_{x \rightarrow 3^+} \frac{2}{x-3} = \underline{\hspace{2cm}} \quad B) \lim_{x \rightarrow 3^-} \frac{2}{x-3} = \underline{\hspace{2cm}} \quad C) \lim_{x \rightarrow 3} \frac{2}{x-3} = \underline{\hspace{2cm}}$$

$$\text{EX2) } \lim_{x \rightarrow 3} \frac{2}{(x-3)^2}$$

$$\text{EX3) } \lim_{x \rightarrow (\pi/2)^+} -2\sec x$$

## Properties of Infinite Limits

**GIVEN:**  $\lim_{x \rightarrow c} f(x) = \infty$  and  $\lim_{x \rightarrow c} g(x) = L$  (some finite limit)

$$\lim_{x \rightarrow c} f(x) \pm g(x) = \infty$$

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \pm\infty \quad \text{and} \quad \lim_{x \rightarrow c} \frac{g(x)}{f(x)} = 0$$

$$\lim_{x \rightarrow c} f(x) \cdot g(x) = \pm\infty$$

**Indeterminate Forms:** **Cannot** determine limit based on any of the following

$$0 \cdot \infty \quad \infty - \infty \quad \frac{0}{0} \quad \frac{\infty}{\infty} \quad 1^\infty \quad \infty^0 \quad 0^0$$

When evaluating infinite limits without a calculator, use your knowledge of graphs along with the properties of limits and infinite limits.

EX)  $\lim_{x \rightarrow 0} 3 + \frac{2}{x^2}$

EX)  $\lim_{x \rightarrow -1^+} \frac{x+2}{x+1}$

EX)  $\lim_{x \rightarrow 4^-} \frac{1}{x^2 - 16}$

EX)  $\lim_{x \rightarrow 0^+} \frac{|x|}{x}$