Integration by Parts

Integration by Parts: Method of integrating PRODUCTS(since there is NO product rule)

Proof:

$$\frac{d}{dx}u \bullet v = u\frac{dv}{dx} + v\frac{du}{dx}$$ (product rule for differentiation)

$$\int \frac{d}{dx}u \bullet v = \int u\frac{dv}{dx} + \int v\frac{du}{dx}$$

$$u \bullet v = \int u\frac{dv}{dx} + \int v\frac{du}{dx}$$

$$u \bullet v - \int v\frac{du}{dx} = \int u\frac{dv}{dx}$$

Formula for Integration by Parts

$$\int u\frac{dv}{dx} = u \bullet v - \int v\frac{du}{dx}$$

where u and v are functions with respect to x and dv, du are the derivatives of v & u with respect to x.

How to integrate using integration by parts:

1) Pick one part of the integral to equal u and the rest to equal dv

2) Differentiate the equation containing "u"

3) Integrate the equation containing "dv"

4) Use the formula above and re-substitute contents from step #3 & #4. Hopefully, the integral remaining can now be completed otherwise integration by parts must be done again.

Products of ***polynomials*** and ***exponential/trigonometric*** functions

Let u= the polynomial and dv = the exponential/trigonometric functions

EX) $\int xe^x \, dx$

EX) $\int x\sin x \, dx$

Note: if polynomial contains degree of two or higher, integration by parts will be needed on the integral remaining until only dx remains from its derivative!

EX) $\int x^2e^x \, dx$

Continue to follow the rule of letting "u" equal any logarithm or inverse trigonometric function

EX) $\int x^2 \ln x \, dx$

EX) $\int_0^1 \arcsin x \, dx$

Products of ***exponential*** functions and ***trigonometric*** functions

Let u = the trigonometric function and dv = the exponential function

"Tricky Finish"

EX) $\int e^x \sin x \, dx$