1a) $\int xe^{-4x}\,dx$

1b) $\int x^3e^x\,dx$

1c) $\int (x^2-1)e^x\,dx$

$$dv = e^{-4x}\,dx \Rightarrow v = \int e^{-4x}\,dx = -\frac{1}{4}e^{-4x}$$

$$u = x \qquad \Rightarrow du = dx$$

$$\int xe^{-4x}\,dx = x\left(-\frac{1}{4}e^{-4x}\right) - \int -\frac{1}{4}e^{-4x}\,dx$$

$$= -\frac{x}{4}e^{-4x} - \frac{1}{16}e^{-4x} + C$$

$$= -\frac{1}{16e^{4x}}(1+4x) + C$$

2a) $\int t\ln(t+1)\,dt$

2b) $\int x^3 \ln x\,dx$

$$dv = t\,dt \quad \Rightarrow \quad v = \int t\,dt = \frac{t^2}{2}$$

$$u = \ln(t+1) \Rightarrow du = \frac{1}{t+1}\,dt$$

$$\int t\ln(t+1)\,dt = \frac{t^2}{2}\ln(t+1) - \frac{1}{2}\int\frac{t^2}{t+1}\,dt$$

$$= \frac{t^2}{2}\ln(t+1) - \frac{1}{2}\int\left(t-1+\frac{1}{t+1}\right)dt$$

$$= \frac{t^2}{2}\ln(t+1) - \frac{1}{2}\left[\frac{t^2}{2} - t + \ln(t+1)\right] + C$$

$$= \frac{1}{4}\left[2(t^2-1)\ln|t+1| - t^2 + 2t\right] + C$$

3a) $\int x^3 \sin x \, dx$

Use integration by parts three times.

(1) $u = x^3, du = 3x^2\,dx, dv = \sin x\,dx, v = -\cos x$

$$\int x^3 \sin dx = -x^3 \cos x + 3\int x^2 \cos x\,dx$$

(2) $u = x^2, du = 2x\,dx, dv = \cos x\,dx, v = \sin x$

$$\int x^3 \sin x\,dx = -x^3 \cos x + 3\left(x^2 \sin x - 2\int x \sin x\,dx\right) = -x^3 \cos x + 3x^2 \sin x - 6\int x \sin x\,dx$$

(3) $u = x, du = dx, dv = \sin x\,dx, v = -\cos x$

$$\begin{aligned}\int x^3 \sin x\,dx &= -x^3 \cos x + 3x^2 \sin x - 6\left(-x \cos x + \int \cos x\,dx\right)\\ &= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C\\ &= \left(6x - x^3\right) \cos x + \left(3x^2 - 6\right) \sin x + C\end{aligned}$$

3b) $\int x \cos x \, dx$

3c) $\int t \csc t \cot t \, dt$

4a) $\int \arctan x \, dx$

4b) $\int_0^{1/2} \arccos x \, dx$

$dv = dx \quad \Rightarrow \quad v = \int dx = x$

$u = \arctan x \quad \Rightarrow \quad du = \dfrac{1}{1 + x^2}\,dx$

$$\begin{aligned}\int \arctan x\,dx &= x \arctan x - \int \frac{x}{1 + x^2}\,dx\\ &= x \arctan x - \frac{1}{2}\ln\left(1 + x^2\right) + C\end{aligned}$$

5a) $\int e^{2x} \sin x \, dx$

Use integration by parts twice.

(1) $dv = e^{2x}\,dx \Rightarrow v = \int e^{2x}\,dx = \frac{1}{2}e^{2x}$

$u = \sin x \Rightarrow du = \cos x\,dx$

(2) $dv = e^{2x}\,dx \Rightarrow v = \int e^{2x}\,dx = \frac{1}{2}e^{2x}$

$u = \cos x \Rightarrow du = -\sin x\,dx$

$$\int e^{2x} \sin x\,dx = \frac{1}{2}e^{2x} \sin x - \frac{1}{2}\int e^{2x} \cos x\,dx = \frac{1}{2}e^{2x} \sin x - \frac{1}{2}\left(\frac{1}{2}e^{2x} \cos x + \frac{1}{2}\int e^{2x} \sin x\,dx\right)$$

$$\frac{5}{4}\int e^{2x} \sin x\,dx = \frac{1}{2}e^{2x} \sin x - \frac{1}{4}e^{2x} \cos x$$

$$\int e^{2x} \sin x\,dx = \frac{1}{5}e^{2x}(2 \sin x - \cos x) + C$$

5b) $\int e^{-x} \cos 2x \, dx$

5c) $\int_0^1 e^{x} \sin x \, dx$

6) AP MULTIPLE CHOICE EXAMPLES

1) $\int \ln 2x \, dx =$

(A) $\dfrac{\ln 2x}{2x} + C$

(B) $x \ln x - x + C$

(C) $x \ln 2x - x + C$

(D) $2x \ln 2x - 2x + C$

2) $\int \cos x \ \ln(\sin x) \, dx =$

(A) $-\cos x \ \ln(\sin x) - \cos x + C$

(B) $-\cos x \ \ln(\sin x) + \sin x + C$

(C) $\sin x \ \ln(\sin x) - \sin x + C$

(D) $\sin x \ \ln(\sin x) + \sin x + C$

3) $\int_0^2 xe^x \, dx =$

(A) $e^2 - 1$ (B) $e^2 + 1$ (C) $e - 1$ (D) $e + 1$

4) $\int_0^{\pi/4} x \sec^2 x \, dx =$

(A) $\dfrac{\pi}{4} - \ln 2$ (B) $\dfrac{\pi}{4} + \ln 2$ (C) $\dfrac{\pi}{4} - \dfrac{\ln 2}{2}$ (D) $\dfrac{\pi}{4} + \dfrac{\ln 2}{2}$