1b) Use integration by parts three times.

(1) $dv = e^x\,dx \Rightarrow \quad v = \int e^x\,dx = e^x$

$u = x^3 \quad \Rightarrow du = 3x^2\,dx$

(2) $dv = e^x\,dx \Rightarrow \quad v = \int e^x\,dx = e^x$

$u = x^2 \quad \Rightarrow du = 2x\,dx$

(3) $dv = e^x\,dx \Rightarrow \quad v = \int e^x\,dx = e^x$

$u = x \quad \Rightarrow du = dx$

$$\int x^3 e^x\,dx = x^3e^x - 3\int x^2e^x\,dx = x^3e^x - 3x^2e^x + 6\int xe^x\,dx$$

$$= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$$

1c) Use integration by parts twice.

(1) $dv = e^x\,dx \Rightarrow \quad v = \int e^x\,dx = e^x$

$u = x^2 \quad \Rightarrow du = 2x\,dx$

(2) $dv = e^x\,dx \Rightarrow \quad v = \int e^x\,dx = e^x$

$u = x \quad \Rightarrow du = dx$

$$\int (x^2 - 1)e^x\,dx = \int x^2e^x\,dx - \int e^x\,dx = x^2e^x - 2\int xe^x\,dx - e^x$$

$$= x^2e^x - 2\left(xe^x - \int e^x\,dx\right) - e^x = x^2e^x - 2xe^x + e^x + C$$

2b) $dv = x^3\,dx \Rightarrow \quad v = \int x^3\,dx = \frac{x^4}{4}$

$u = \ln x \quad \Rightarrow du = \frac{1}{x}\,dx$

$$\int x^3 \ln x\,dx = \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3\,dx = \frac{x^4}{4}\ln x - \frac{x^4}{16} + C$$

$$= \frac{x^4}{16}(4\ln x - 1) + C$$

3b) $dv = \cos x\,dx \quad \Rightarrow \quad v = \int \cos x\,dx = \sin x$

$u = x \quad \Rightarrow \quad du = dx$

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C$$

3c) $u = t,\ du = dt,\ dv = \csc t\cot dt,\ v = -\csc t$

$$\int t\csc t\cot t\,dt = -t\csc t + \int \csc t\,dt = -t\csc t - \ln\left|\csc t + \cot t\right| + C$$

4b) $u = \arccos x,\ du = -\frac{1}{\sqrt{1-x^2}}\,dx,\ dv = dx,\ v = x$

$$\int \arccos x\,dx = x\arccos x + \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$= x\arccos x - \sqrt{1-x^2} + C$$

So,

$$\int_0^{1/2} \arccos x = \left[x\arccos x - \sqrt{1-x^2}\right]_0^{1/2}$$

$$= \frac{1}{2}\arccos\left(\frac{1}{2}\right) - \sqrt{\frac{3}{4}} + 1$$

$$= \frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1 \approx 0.658.$$

5b) Use integration by parts twice.

(1) $dv = e^{-x}\,dx \;\Rightarrow\; v = \int e^{-x}\,dx = -e^{-x}$

$u = \cos 2x \;\Rightarrow\; du = -2\sin 2x\,dx$

$$\int e^{-x}\cos 2x\,dx = \cos 2x\left(-e^{-x}\right) - \int\left(-e^{-x}\right)\left(-2\sin 2x\right)dx = -e^{-x}\cos 2x - 2\int e^{-x}\sin 2x\,dx$$

(2) $dv = e^{-x}\,dx \;\Rightarrow\; v = \int e^{-x}\,dx = -e^{-x}$

$u = \sin 2x \;\Rightarrow\; du = 2\cos 2x\,dx$

$$\int e^{-x}\cos 2x\,dx = -e^{-x}\cos 2x - 2\left[\sin 2x\left(-e^{-x}\right) - \int\left(-e^{-x}\right)\left(2\cos 2x\right)dx\right]$$

$$= -e^{-x}\cos 2x + 2e^{-x}\sin 2x - 4\int e^{-x}\cos 2x\,dx$$

$$(4+1)\int e^{-x}\cos 2x\,dx = -e^{-x}\cos 2x + 2e^{-x}\sin 2x$$

$$\int e^{-x}\cos 2x\,dx = \tfrac{1}{5}e^{-x}\left(2\sin 2x - \cos 2x\right) + C$$

5c) Use integration by parts twice.

(1) $dv = e^{x}\,dx \;\Rightarrow\; v = \int e^{x}\,dx = e^{x}$ (2) $dv = e^{x}\,dx \;\Rightarrow\; v = \int e^{x}\,dx = e^{x}$

$u = \sin x \;\Rightarrow\; du = \cos x\,dx$ $u = \cos x \;\Rightarrow\; du = -\sin x\,dx$

$$\int e^{x}\sin x\,dx = e^{x}\sin x - \int e^{x}\cos x\,dx = e^{x}\sin x - e^{x}\cos x - \int e^{x}\sin x\,dx$$

$$2\int e^{x}\sin x\,dx = e^{x}\left(\sin x - \cos x\right)$$

$$\int e^{x}\sin x\,dx = \frac{e^{x}}{2}\left(\sin x - \cos x\right) + C$$

So, $\int_0^1 e^{x}\sin x\,dx = \left[\frac{e^{x}}{2}\left(\sin x - \cos x\right)\right]_0^1 = \frac{e}{2}\left(\sin 1 - \cos 1\right) + \frac{1}{2} = \frac{e\left(\sin 1 - \cos 1\right) + 1}{2} \approx 0.909$

1) $\int \ln 2x \, dx =$ $u \cdot v - \int v \, du$

$u = \ln(2x)$ $dv = 1 dx$

$\frac{du}{dx} = \frac{1}{2x} \cdot 2$ $\int dv = \int 1 dx$

$du = \frac{1}{x} dx$ $v = x$

$x \cdot \ln(2x) - \int x \cdot \frac{1}{x} dx$

$x \ln(2x) - \int 1 dx$

$x \ln(2x) - x + C$

(A) $\frac{\ln 2x}{2x} + C$

(B) $x \ln x - x + C$

(C) $x \ln 2x - x + C$

(D) $2x \ln 2x - 2x + C$

2) $\int \cos x \ln(\sin x) \, dx =$ $u \cdot v - \int v \, du$

$u = \ln(\sin x)$ $dv = \cos x \, dx$

$\frac{du}{dx} = \frac{1}{\sin x} \cdot \cos x$ $\int dv = \int \cos x \, dx$

$v = \sin x$

$du = \cot x \, dx$

(A) $-\cos x \ln(\sin x) - \cos x + C$

(B) $-\cos x \ln(\sin x) + \sin x + C$

(C) $\sin x \ln(\sin x) - \sin x + C$

(D) $\sin x \ln(\sin x) + \sin x + C$

$\sin x \cdot \ln(\sin x) - \int \sin x \cdot \cot x \, dx$

$\sin x \ln(\sin x) - \int \cos x \, dx = \sin x \cdot \ln(\sin x) - \sin x + C$

3) $\int_0^2 xe^x \, dx =$ $u \cdot v - \int v \, du$

$u = x$ $dv = e^x dx$

$\frac{du}{dx} = 1$ $\int dv = \int e^x dx$

$v = e^x$

(A) $e^2 - 1$ (B) $e^2 + 1$ (C) $e - 1$ (D) $e + 1$

$= xe^x - \int e^x dx$

$= xe^x - e^x \Big|_0^2 = (2e^2 - e^2) - (0e^0 - e^0) = e^2 + 1$

4) $\int_0^{\pi/4} x \sec^2 x \, dx =$ $u \cdot v - \int v \, du$

$u = x$ $dv = \int$

$\frac{du}{dx} = 1$

(A) $\frac{\pi}{4} - \ln 2$ (B) $\frac{\pi}{4} + \ln 2$ (C) $\frac{\pi}{4} - \frac{\ln 2}{2}$ (D) $\frac{\pi}{4} + \frac{\ln 2}{2}$

$u = x$ $dv = \sec^2 x \, dx$

$\frac{du}{dx} = 1$ $\int dv = \int \sec^2 x \, dx$

$du = dx$ $v = \tan x$

$= x \tan x - \int \tan x \, dx$

$= x \tan x + \ln|\cos x| \Big|_0^{\frac{\pi}{4}}$

$= \left(\frac{\pi}{4} \cdot \tan\frac{\pi}{4} + \ln\left|\cos\frac{\pi}{4}\right|\right) - (0 + \ln|\cos(0)|)$

$= \left(\frac{\pi}{4} + \ln\frac{\sqrt{2}}{2}\right) - (0 + \ln 1)$

$= \left(\frac{\pi}{4} + \ln\frac{\sqrt{2}}{2}\right) - 0$

$= \frac{\pi}{4} - \frac{1}{2}\ln 2$

$\ln\frac{\sqrt{2}}{2} = \ln\frac{1}{\sqrt{2}} = \ln 1 - \ln\sqrt{2}$

$= 0 - \ln 2^{1/2}$

$= -\frac{1}{2}\ln 2$