

## Integration by Substitution (u-substitution)

W-up: Evaluate  $\int_1^3 (2t-4)^3 dt$  without a calculator

**U-Substitution:** used to integrate the product, quotient or composition of functions(that can't be easily simplified into singular powers of the variable)

Examples of Integrals where U-substitution is needed:

$$\int (t+2)^5 dt \quad \int_3^6 \sqrt{(2t-4)} dt \quad \int \sin(2x+1)dx \quad \int_0^\pi \tan x \sec^2 x dx \quad \int \frac{x-1}{x+1} dx$$

**EX)**  $\int x^2 \sqrt{x^3+1} dx$

$$\int x^2 \sqrt{u} dx$$

$u = x^3 + 1$ $\frac{du}{dx} = 3x^2$ $\text{so...} dx = \frac{du}{3x^2}$
--

$$\int x^2 \sqrt{u} \frac{du}{3x^2}$$

$$\int \frac{1}{3} \sqrt{u} du$$

$$\frac{2}{9} u^{3/2} + C$$

$$\frac{2}{9} (x^3 + 1)^{3/2} + C$$

1) Allow something to equal "**u**"(usually the binomial or contents

inside grouping symbols or one of the terms of a product)

2) Differentiate "**u**" with respect to  $x$  and solve for  $dx$ .

Replace the  $dx$  in the integral with the answer(should be an expression with  $u$  and  $du$ )

3) Simplify and integrate if possible

4) Resubstitute **u** with contents in terms of  $x$

$$\text{EX: } \int (2t-4)^3 dt$$

$$\text{EX: } \int_2^9 \sqrt{3x-2} dx$$

$$\text{EX: } \int \frac{x^2}{(16-x^3)^2} dx$$

Note: be sure to evaluate the correct values for  $x$  or convert them to  $u$ 's

**Note:** when substitution does NOT yield an expression in terms of just ONE variable (*variables other than  $u$  remain*) solve your substitution equation for  $x$  and substitute that expression (in terms of  $u$ ) into your integral for  $x$  as well.

$$\text{EX: } \int x\sqrt{x-1} dx$$

## DECISIONS, DECISIONS

Generally:

Let  $u$  be anything (expression) under a radical

Let  $u$  equal the highest power expression

Let  $u$  be the expression in the denominator of a fraction (the binomial part)

**NOTE: DO NOT BE ROBOTIC WHEN INTEGRATING! Some functions look like they need u-substitution, but really just need to be rewritten or simplified first!**

$$\text{EX: } \int (x-1)\sqrt{x} \, dx$$

$$\text{EX: } \int \frac{x^2 - 2x + 3}{\sqrt{x}} \, dx$$

### Substitution with Trigonometric Functions

#### One Trig. Function(or direct reversal)

Let  $u =$  “the angle” and proceed with substitution

$$\text{EX: } \int \sin(2x+1) \, dx$$

$$\text{EX: } \int \sec(3x-4)\tan(3x-4) \, dx$$

#### Multiple Trig. Functions

Let  $u =$  the trig. function whose derivative is the other function and proceed with substitution

$$\text{EX: } \int \tan x \sec^2 x \, dx$$

**Note:** When sine and cosine are used and multiplied together *let  $u =$  the powered trig function!*

$$\text{EX: } \int \sin^2 x \cos x \, dx$$