Logarithmic Functions and Integration

W-up: AP Multiple Choice # 8, 11(both non calculator)

Differentiate each function:

A) $f(x)=\ln(x)$ B) $f(x)=\ln(4x^2-7)$ C) $f(x)=\log_3(2x-5)$

When the power rule is used to integrate, a variable or expression containing a power of "negative one" will not work since we may not divide by zero. This is not a problem, because this is the answer to the derivative of **ln*x*** !

$$\int \frac{1}{u}du=\ln|u|+C$$

Note: absolute value assures us that the domain of "u" in the integral matches the domain of "u" in its logarithmic answer!

Since NO derivative *ever* contains a non-constant logarithmic function, ALWAYS let "u" equal the log function!

EX) $\int \frac{1}{x\ln x}dx$ EX) $\int \frac{\log_3(x-1)}{x-1}dx$ EX) $\int \frac{x-1}{x}dx$

Integrating Rational Expressions(denominator not a monomial)

Degree of numerator is one less than the denominator

Let "u" = the denominator

EX) $\int \frac{1}{5x+3}dx$

EX) $\int \frac{3x^2+1}{x^3+x}dx$

Degree of numerator is greater than or equal to the denominator

EX) $\int \frac{x^2+3x+1}{x+1}dx$

Long Division

$\int x+2-\frac{1}{x+1}dx$

$\int x\,dx+\int 2\,dx-\int \frac{1}{x+1}\,dx$

Now integrate the first two integrals from the past and follow the strategy above for the last integral

Re-substitution

$\int \frac{x^2+3x+1}{u}du$ $\quad u=x+1$, $du=dx$

Since *x*'s remain, write them in terms of *u*!

If $u=x+1$ **then** $x=u-1$

$\int \frac{(u-1)^2+3(u-1)+1}{u}du$

$\int \frac{u^2+u-1}{u}du$

$\int u+1-\frac{1}{u}du$ .....

EX) $\int \frac{2x}{(x+1)^2}dx$

EX) $\int \frac{1}{\sqrt{x}+1}dx$

Use u-substitution to integrate $\int \frac{\sin x}{\cos x}dx$

Since we know $\int \sin x\ dx$ and $\int \cos\ dx$ we can use u-substitution to find the integrals of the other four trig. functions

$$\int \tan u\ du = -\ln|\cos u| + C$$

$$\int \cot u\ du = \ln|\sin u| + C$$

$$\int \sec u\ du = \ln|\sec u + \tan u| + C$$

$$\int \csc u\ du = -\ln|\csc u + \cot u| + C$$

EX) $\int \cot\frac{x}{2}dx$

EX) $\int_0^{\pi/4} \sqrt{1+\tan^2 x}\ dx$