Inverse Trigonometric Functions and Integration

W-up: AP Multiple Choice # 2(non-calculator)

Differentiate each function:

A) $f(x)=\text{Arcsin}(3x)$ B) $f(x)=\text{Cot}^{-1}(2x+5)$ C) $f(x)=\text{Arcsec}(2x)$

Since NO derivative *ever* contains an inverse trigonometric function, ALWAYS let "u" equal the inverse trig. function!

EX) $\int \frac{\arccos x}{\sqrt{1-x^2}}dx$

Integrals of Inverse Trigonometric Functions

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du=\text{Arcsin}\left(\frac{u}{a}\right)+C$$

$$\int \frac{1}{a^2+u^2}\,du=\frac{1}{a}\text{Arctan}\left(\frac{u}{a}\right)+C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du=\frac{1}{a}\text{Arcsec}\left(\frac{|u|}{a}\right)+C$$

Where "*a*" is a constant and "*u*" is an algebraic expression

EX) $\int \frac{1}{9x^2+3}dx$

EX) $\int \frac{1}{\sqrt{e^{2x}-1}}\,dx$

Decisions, Decisions…

EX) $\int \frac{x}{x^2+9}dx$

EX) $\int \frac{10}{x^2+9}dx$

EX) $\int \frac{x+10}{x^2+9}dx$

EX) $\int \frac{x^2+10}{x^2+9}dx$

Completing the Square can be used for quadratic equations in the denominator with a LINEAR term.

EX) $\int \frac{1}{x^2-4x+8}dx$

EX) $\int \frac{3}{2x^2-8x+10}dx$

EX) $\int \frac{x}{\sqrt{-x^4+8x^2+9}}dx$