AP Calculus Differentiation of Inverse Functions Critical Homework Answers

1b) $f(x) = x^3 + 2x - 1, \quad a = 2$

$f'(x) = 3x^2 + 2 > 0$

$f(1) = 2 \Rightarrow f^{-1}(2) = 1$

$$\left(f^{-1}\right)'(2) = \frac{1}{f'\left(f^{-1}(2)\right)} = \frac{1}{f'(1)} = \frac{1}{3\left(1^2\right) + 2} = \frac{1}{5}$$

1c) $f'(x) = \cos x > 0 \text{ on } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$f\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2} \Rightarrow f^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$$\left(f^{-1}\right)'\left(\frac{1}{2}\right) = \frac{1}{f'\left(f^{-1}\left(\frac{1}{2}\right)\right)}$$

$$= \frac{1}{f'\left(\frac{\pi}{6}\right)} = \frac{1}{\cos\left(\frac{\pi}{6}\right)} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

1d) $f(x) = \frac{x + 6}{x - 2}, \quad x > 0, a = 3$

$$f'(x) = \frac{(x-2)(1) - (x+6)(1)}{(x-2)^2}$$

$$= \frac{-8}{(x-2)^2}$$

$f(6) = 3 \Rightarrow f^{-1}(3) = 6$

$$\left(f^{-1}\right)'(3) = \frac{1}{f'\left(f^{-1}(3)\right)} = \frac{1}{f'(6)} = \frac{1}{-8/(6-2)^2} = -2$$

2b) $x = 2\ln(y^2 - 3)$

$$1 = 2 \bullet \frac{1}{y^2 - 3} \bullet 2y\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y^2 - 3}{4y}$$

$$\frac{dy}{dx}(0,2) = \frac{1}{8}$$

3a) If (2, 0) is on $g(x)$ then (0, 2) is on $f(x)$.

$f'(x) = 3x^2 + 4$

$$g'(2) = \frac{1}{f'(0)}$$

$$= \frac{1}{4}$$