

Given $f \circ g(x) = x$, use implicit differentiation to find derivative of $g(x)$ at the given point.

1a) $f(x) = x^3 - 7x^2 + 2$ and $g(-4) = 1$

$$x = y^3 - 7y^2 + 2$$

$$1 = 3y^2 \frac{dy}{dx} - 14y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3y^2 - 14y}$$

$$\text{At } (-4, 1), \frac{dy}{dx} = \frac{1}{3 - 14} = \frac{-1}{11}$$

1b) $f(x) = 2\ln(x^2 - 3)$ and $g(0) = 2$

$$\text{INVERSE} \Rightarrow x = 2 \ln(y^2 - 3)$$

$$1 = 2 \cdot \frac{1}{y^2 - 3} \cdot 2y \frac{dy}{dx}$$

$$1 = \frac{4y}{y^2 - 3} \frac{dy}{dx}$$

$$\begin{aligned} \text{deriv.} &\Rightarrow \frac{y^2 - 3}{4y} \\ \text{inverse} &= \frac{dy}{dx} \end{aligned}$$

$$\frac{2^2 - 3}{4(2)} = \frac{dy}{dx}(0, 2)$$

$$\boxed{\frac{1}{8}} = g'(0)$$

2) AP MULTIPLE CHOICE EXAMPLES

- 1) If the function f is defined by $f(x) = x^5 - 1$, then f^{-1} , the inverse function of f , is defined by

$$f^{-1}(x) =$$

$$\text{INV} \Rightarrow y = x^5 - 1$$

$$(A) \quad \frac{1}{\sqrt[5]{x+1}}$$

$$\sqrt[5]{x+1} = y \quad (B) \quad \frac{1}{\sqrt[5]{x+1}}$$

$$(C) \quad \sqrt[5]{x-1}$$

$$(D) \quad \sqrt[5]{x-1}$$

$$(E) \quad \sqrt[5]{x+1}$$

(E) $\sqrt[5]{x+1}$

- 2) Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and

$$\text{if } g(-2) = 5 \text{ and } f'(5) = -\frac{1}{2}, \text{ then } g'(-2) =$$

slope of $g(x)$ at $(-2, 5)$

$$(A) \quad 2$$

$$(B) \quad \frac{1}{2}$$

$$(C) \quad \frac{1}{5}$$

$$(D) \quad -\frac{1}{5}$$

$$(E) \quad -2$$

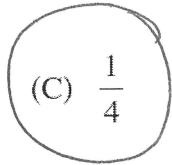
$g(x)$ contains $(-2, 5)$

$f(x)$ contains $(5, -2)$

if $f'(5) = -\frac{1}{2}$ then $g'(-2) = -2$

- 3) The function f is defined by $f(x) = x^3 + 4x + 2$. If g is the inverse function of f and $g(2) = 0$, what is the value of $g'(2)$?

(A) $-\frac{1}{16}$ (B) $-\frac{4}{81}$ (C) $\frac{1}{4}$ (D) 4



$g(x)$ contains $(2, 0)$ ← $g'(2)$ is slope at this point

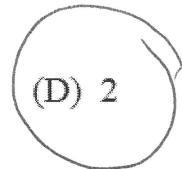
$f(x)$ contains $(0, 2)$ ← slope of this point is 4

$$f'(x) = 3x^2 + 4$$

$$f'(0) = 4$$

- 4) If $f(x) = \sin x$, then $(f^{-1})'(\frac{\sqrt{3}}{2}) =$

(A) $\frac{1}{2}$ (B) $\frac{2\sqrt{3}}{3}$ (C) $\sqrt{3}$



$$f^{-1}(x) = \arcsin x$$

$$[f^{-1}(x)]' = \frac{1}{\sqrt{1-x^2}}$$

$$[f^{-1}(\frac{\sqrt{3}}{2})]' = \frac{1}{\sqrt{1-(\frac{\sqrt{3}}{2})^2}}$$

$$= \frac{1}{\sqrt{1-\frac{3}{4}}} \\ = \frac{1}{\sqrt{\frac{1}{4}}} \\ = \frac{1}{\frac{1}{2}}$$

$$= -\frac{1}{\frac{1}{2}}$$

$$= 2$$

$f^{-1}(x)$ contains $(\frac{\sqrt{3}}{2}, ?)$

$f(x)$ contains $(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$

must be
 $\frac{\pi}{3}$ since
 $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

OR $f'(x) = \cos x$

$$f'(\frac{\pi}{3}) = \frac{1}{2}$$

$$\text{so.. } [f^{-1}(\frac{\sqrt{3}}{2})]' = 2$$