

Find the derivative of each function.

1a)  $g(x) = 3 \arccos \frac{x}{2}$

$$g'(x) = \frac{-3(1/2)}{\sqrt{1 - (x^2/4)}} = \frac{-3}{\sqrt{4 - x^2}}$$

Note:  $\sqrt{1 - \frac{x^2}{4}} = \sqrt{\frac{4 - x^2}{4}} = \frac{\sqrt{4 - x^2}}{2}$

1b)  $f(x) = 2 \arcsin(x - 1)$

$$f'(x) = 2 \cdot \frac{1}{\sqrt{1 - (x-1)^2}} \cdot 1$$

$$f'(x) = \frac{2}{\sqrt{1 - (x^2 - 2x + 1)}}$$

$$f'(x) = \frac{2}{\sqrt{2x - x^2}}$$

1c)  $f(x) = \arctan(e^x)$

$$f'(x) = \frac{1}{1 + (e^x)^2} \cdot e^x$$

$$f'(x) = \frac{e^x}{1 + e^{2x}}$$

1d)  $g(x) = \frac{\arcsin 3x}{x}$

$$g'(x) = \frac{x \cdot \frac{1}{\sqrt{1-9x^2}} \cdot 3 - \arcsin x}{x^2}$$

$$g'(x) = \frac{3x}{\sqrt{1-9x^2}} - \arcsin x$$

$$g'(x) = \frac{3x - \sqrt{1-9x^2} \arcsin x}{x^2 \sqrt{1-9x^2}}$$

1e)  $f(x) = \arcsin x + \arccos x$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}}$$

$$f'(x) = 0$$

2a)  $y = \arcsin x + x\sqrt{1-x^2}$

$$y' = \frac{1}{\sqrt{1-x^2}} + x \left( \frac{1}{2} \right) (-2x)(1-x^2)^{-1/2} + \sqrt{1-x^2}$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2}$$

$$= \sqrt{1-x^2} + \sqrt{1-x^2} = 2\sqrt{1-x^2}$$

$$y' = \frac{-2 - 2x^2 + 2x^2 - 2}{4x^2 - 4}$$

$$y' = \frac{-4}{4(x^2-1)}$$

$$y' = \frac{-1}{(x+1)(x-1)}$$

2b)  $y = \frac{1}{2} \left( \frac{1}{2} \ln \frac{x+1}{x-1} + \arctan x \right)$

$$y = \frac{1}{4} \ln(x+1) - \frac{1}{4} \ln(x-1) + \frac{1}{2} \arctan x$$

$$y' = \frac{1}{4} \cdot \frac{1}{x+1} - \frac{1}{4} \cdot \frac{1}{x-1} + \frac{1}{2} \cdot \frac{1}{1+x^2}$$

$$y' = \frac{1}{4x+4} - \frac{1}{4x-4} + \frac{1}{2+2x^2}$$

$$y' = \frac{4x-4 - (4x+4)}{16x^2-16} + \frac{1}{2+2x^2}$$

$$y' = \frac{-8}{16(x^2-1)} + \frac{1}{2+2x^2}$$

$$y' = \frac{-1}{2x^2-2} + \frac{1}{2+2x^2}$$

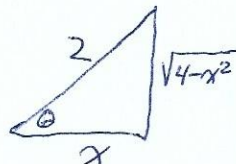
3a)  $h(t) = \sin(\arccos t)$

$$h(t) = \sin(\arccos t) = \sqrt{1-t^2}$$

$$h'(t) = \frac{1}{2}(1-t^2)^{-1/2}(-2t)$$

$$= \frac{-t}{\sqrt{1-t^2}}$$

3b)  $f(x) = \tan\left(\arccos \frac{x}{2}\right)$



$$\tan(\arccos \frac{x}{2}) = \frac{\sqrt{4-x^2}}{x}$$

$\theta \rightarrow$  angle whose cosine equals " $\frac{x}{2}$ "

So,  $f(x) = \frac{(4-x^2)^{1/2}}{x}$

$$f'(x) = \frac{x \cdot \frac{1}{2}(4-x^2)^{-1/2} \cdot (-2x) - (4-x^2)^{1/2} \cdot 1}{x^2}$$

$$f'(x) = \frac{\left(\frac{-x^2}{\sqrt{4-x^2}} - \sqrt{4-x^2}\right)}{x^2} \cdot \frac{\sqrt{4-x^2}}{\sqrt{4-x^2}} = \frac{-x^2 - (4-x^2)}{x^2 \sqrt{4-x^2}}$$

$$f'(x) = \frac{-4}{x^2 \sqrt{4-x^2}}$$

Find the equation of the tangent line at the given point of each equation.

4a)  $y = \arctan \frac{x}{2}, \quad \left(2, \frac{\pi}{4}\right)$

$$y' = \frac{1}{1+(x^2/4)} \left(\frac{1}{2}\right) = \frac{2}{4+x^2}$$

At  $\left(2, \frac{\pi}{4}\right), y' = \frac{2}{4+4} = \frac{1}{4}$

Tangent line:  $y - \frac{\pi}{4} = \frac{1}{4}(x - 2)$

4b)  $y = 4x \arccos(x-1), \quad (1, 2\pi)$

$$y' = 4x \cdot \frac{-1}{\sqrt{1-(x-1)^2}} \cdot 1 + 4 \cdot \arccos(x-1)$$

$$y'(1) = \frac{-4}{1} + 4 \cdot \frac{\pi}{2}$$

$$y'(1) = -4 + 2\pi$$

So... 
$$\begin{aligned} y - 2\pi &= (-4 + 2\pi)(x - 1) \\ \text{or} \\ y &= -4x + 4 + 2\pi x - 2\pi + 2\pi \end{aligned}$$

4c)  $x^2 + x \arctan y = y - 1, \quad \left(-\frac{\pi}{4}, 1\right)$

$$2x + x \cdot \frac{1}{1+y^2} \frac{dy}{dx} + \arctan y \cdot 1 = 1 \cdot \frac{dy}{dx}$$

$$2x + \arctan y = 1 \frac{dy}{dx} - \frac{x}{1+y^2} \frac{dy}{dx}$$

$$2x + \arctan y = \frac{dy}{dx} \left(1 - \frac{x}{1+y^2}\right)$$

$$\frac{2x + \arctan y}{1 - \frac{x}{1+y^2}} = \frac{dy}{dx}$$

$$\frac{-\frac{\pi}{2} + \frac{\pi}{4}}{1 - \frac{-\pi/4}{2}} = \frac{dy}{dx} \left(-\frac{\pi}{4}, 1\right)$$

$$\frac{dy}{dx} \left(-\frac{\pi}{4}, 1\right) = \frac{\left(-\frac{\pi}{2} + \frac{\pi}{4}\right)}{\left(1 + \frac{\pi}{8}\right)} \cdot \frac{8}{8}$$

$$\begin{aligned} \frac{dy}{dx} \left(-\frac{\pi}{4}, 1\right) &= \frac{-4\pi + 2\pi}{8 + \pi} \\ &= \frac{-2\pi}{8 + \pi} \end{aligned}$$

So...

$$y - 1 = \frac{-2\pi}{8 + \pi} \left(x + \frac{\pi}{4}\right)$$

## 5) AP MULTIPLE CHOICE EXAMPLES

1) If  $y = \arctan(\cos x)$ , then  $\frac{dy}{dx} = \frac{1}{1 + (\cos x)^2} \cdot (-\sin x)$

(A)  $\frac{-\sin x}{1 + \cos^2 x}$

(B)  $-(\operatorname{arcsec}(\cos x))^2 \sin x$

(C)  $(\operatorname{arcsec}(\cos x))^2$

(D)  $\frac{1}{(\arccos x)^2 + 1}$

(E)  $\frac{1}{1 + \cos^2 x}$

NOTE: Inverse trig function on OUTSIDE so not one we need to simplify first (using a triangle)

2)  $\frac{d}{dx}(\arcsin 2x) = \frac{1}{\sqrt{1 - (2x)^2}} \cdot 2$

(A)  $\frac{-1}{2\sqrt{1 - 4x^2}}$

(B)  $\frac{-2}{\sqrt{4x^2 - 1}}$

(C)  $\frac{1}{2\sqrt{1 - 4x^2}}$

(D)  $\frac{2}{\sqrt{1 - 4x^2}}$

(E)  $\frac{2}{\sqrt{4x^2 - 1}}$