Find the derivative of each function.

1a) $g(x) = 3\arccos\dfrac{x}{2}$

$$g'(x) = \frac{-3(1/2)}{\sqrt{1-(x^2/4)}} = \frac{-3}{\sqrt{4-x^2}}$$

Note: $\sqrt{1-\dfrac{x^2}{4}} = \sqrt{\dfrac{4-x^2}{4}} = \dfrac{\sqrt{4-x^2}}{2}$

1b) $f(x) = 2\arcsin(x-1)$

$$f'(x) = 2\cdot\frac{1}{\sqrt{1-(x-1)^2}}\cdot 1$$

$$f'(x) = \frac{2}{\sqrt{1-(x^2-2x+1)}}$$

$$f'(x) = \frac{2}{\sqrt{2x-x^2}}$$

1c) $f(x) = \arctan(e^x)$

$$f'(x) = \frac{1}{1+(e^x)^2}\cdot e^x$$

$$\boxed{f'(x) = \frac{e^x}{1+e^{2x}}}$$

1d) $g(x) = \dfrac{\arcsin 3x}{x}$

$$g'(x) = \frac{x\cdot\frac{1}{\sqrt{1-9x^2}}\cdot 3 - \arcsin x}{x^2}$$

$$g'(x) = \frac{\frac{3x}{\sqrt{1-9x^2}} - \arcsin x}{x^2}$$

$$g'(x) = \frac{3x - \sqrt{1-9x^2}\cdot\arcsin x}{x^2\sqrt{1-9x^2}}$$

1e) $f(x) = \arcsin x + \arccos x$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}}$$

$$f'(x) = 0$$

2a) $y = \arcsin x + x\sqrt{1-x^2}$

$$y' = \frac{1}{\sqrt{1-x^2}} + x\left(\frac{1}{2}\right)(-2x)(1-x^2)^{-1/2} + \sqrt{1-x^2}$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2}$$

$$= \sqrt{1-x^2} + \sqrt{1-x^2} = 2\sqrt{1-x^2}$$

2b) $y = \dfrac{1}{2}\left(\dfrac{1}{2}\ln\dfrac{x+1}{x-1} + \arctan x\right)$

$$y = \tfrac{1}{4}\ln(x+1) - \tfrac{1}{4}\ln(x-1) + \tfrac{1}{2}\arctan x$$

$$y' = \frac{1}{4}\cdot\frac{1}{x+1} - \frac{1}{4}\cdot\frac{1}{x-1} + \frac{1}{2}\cdot\frac{1}{1+x^2}$$

$$y' = \frac{1}{4x+4} - \frac{1}{4x-4} + \frac{1}{2+2x^2}$$

or

$$y' = \frac{4x-4-(4x+4)}{16x^2-16} + \frac{1}{2+2x^2}$$

$$y' = \frac{-\cancel{8}\,2}{\cancel{16}(x^2-1)} + \frac{1}{2+2x^2}$$

$$y' = \frac{-1}{2x^2-2} + \frac{1}{2+2x^2}$$

or

$$y' = \frac{-2-\cancel{2x^2}+\cancel{2x^2}-2}{4x^4-4}$$

$$y' = \frac{-4}{4(x^4-1)}$$

$$y' = \frac{-1}{(x^2+1)(x^2-1)} = \frac{-1}{(x^2+1)(x+1)(x-1)}$$

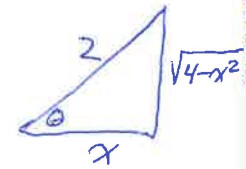
3a) $h(t) = \sin(\arccos t)$

$$h(t) = \sin(\arccos t) = \sqrt{1 - t^2}$$

$$h'(t) = \frac{1}{2}\left(1 - t^2\right)^{-1/2}(-2t)$$

$$= \frac{-t}{\sqrt{1 - t^2}}$$

3b) $f(x) = \tan\left(\arccos \frac{x}{2}\right)$



$\tan\left(\arccos \frac{x}{2}\right) = \frac{\sqrt{4-x^2}}{x}$ $\quad\theta \to$ angle whose cosine equals "$\frac{x}{2}$"

So.. $f(x) = \frac{(4-x^2)^{1/2}}{x}$

$$f'(x) = \frac{x \cdot \frac{1}{\cancel{2}}(4-x^2)^{-1/2} \cdot (-\cancel{2}x) - (4-x^2)^{1/2} \cdot 1}{x^2}$$

$$f'(x) = \frac{\left(\frac{-x^2}{\sqrt{4-x^2}} - \sqrt{4-x^2}\right)}{x^2} \cdot \frac{\sqrt{4-x^2}}{\sqrt{4-x^2}} = \frac{-x^2 - (4-x^2)}{x^2\sqrt{4-x^2}}$$

$$f'(x) = \boxed{\frac{-4}{x^2\sqrt{4-x^2}}}$$

Find the equation of the tangent line at the given point of each equation.

4a) $y = \arctan \frac{x}{2}, \quad \left(2, \frac{\pi}{4}\right)$

$$y' = \frac{1}{1 + (x^2/4)}\left(\frac{1}{2}\right) = \frac{2}{4 + x^2}$$

At $\left(2, \frac{\pi}{4}\right)$, $y' = \frac{2}{4 + 4} = \frac{1}{4}$.

Tangent line: $y - \frac{\pi}{4} = \frac{1}{4}(x - 2)$

4b) $y = 4x \arccos(x - 1), \quad (1, 2\pi)$

$$y' = 4x \cdot \frac{-1}{\sqrt{1-(x-1)^2}} \cdot 1 + 4 \cdot \arccos(x-1)$$

$$y'(1) = \frac{-4}{1} + 4 \cdot \frac{\pi}{2}$$

$$y'(1) = -4 + 2\pi$$

So... $\boxed{y - 2\pi = (-4+2\pi)(x-1)}$ or $\boxed{y = -4x + 4 + 2\pi x \cancel{-2\pi + 2\pi}}$

4c) $x^2 + x \cdot \arctan y = y - 1, \quad \left(-\frac{\pi}{4}, 1\right)$

$$2x + x \cdot \frac{1}{1+y^2}\frac{dy}{dx} + \arctan y \cdot 1 = 1\frac{dy}{dx}$$

$$2x + \arctan y = 1\frac{dy}{dx} - \frac{x}{1+y^2}\frac{dy}{dx}$$

$$2x + \arctan y = \frac{dy}{dx}\left(1 - \frac{x}{1+y^2}\right)$$

$$\frac{2x + \arctan y}{1 - \frac{x}{1+y^2}} = \frac{dy}{dx}$$

$$\frac{-\frac{\pi}{2} + \frac{\pi}{4}}{1 - \frac{-\frac{\pi}{4}}{2}} = \frac{dy}{dx}\left(-\frac{\pi}{4}, 1\right)$$

$$\frac{dy}{dx}\left(-\frac{\pi}{4}, 1\right) = \frac{\left(\frac{-\pi}{2} + \frac{\pi}{4}\right)}{\left(1 + \frac{\pi}{8}\right)} \cdot \frac{8}{8}$$

$$\frac{dy}{dx}\left(-\frac{\pi}{4}, 1\right) = \frac{-4\pi + 2\pi}{8 + \pi}$$

$$= \frac{-2\pi}{8 + \pi}$$

So... $\boxed{y - 1 = \frac{-2\pi}{8+\pi}\left(x + \frac{\pi}{4}\right)}$

5) AP MULTIPLE CHOICE EXAMPLES

1) If $y = \arctan(\cos x)$, then $\frac{dy}{dx} =$ $\frac{1}{1+(\cos x)^2} \cdot (-\sin x)$

(A) $\frac{-\sin x}{1+\cos^2 x}$

(B) $-(\operatorname{arcsec}(\cos x))^2 \sin x$

(C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\frac{1}{(\arccos x)^2+1}$

(E) $\frac{1}{1+\cos^2 x}$

NOTE: Inverse trig function on OUTSIDE so not one we need to simplify first (using a triangle)

2) $\frac{d}{dx}(\arcsin 2x) =$ $\frac{1}{\sqrt{1-(2x)^2}} \cdot 2$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$

(B) $\frac{-2}{\sqrt{4x^2-1}}$

(C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$

(E) $\frac{2}{\sqrt{4x^2-1}}$