

## Differentiation of Inverse Trigonometric Functions

W-up: AP Multiple Choice # 27 (non-calculator)

“ $\text{Arc sin } x$ ” means “the angle whose sine is  $x$ ”

Evaluate

1)  $\text{Arc sin}\left(\frac{1}{2}\right)$

2)  $\text{Cos}^{-1}\left(\frac{1}{2}\right)$

3)  $\text{Arc tan}(\sqrt{3})$

4)  $\text{Arc sin}\left(-\frac{1}{2}\right)$

5)  $\text{Arc cos}\left(-\frac{\sqrt{2}}{2}\right)$

6)  $\text{Arc tan}(-\sqrt{3})$

Remember: The answers to inverse trig functions are ANGLES where

$$-\frac{\pi}{2} \leq \text{Arc sin } x \leq \frac{\pi}{2}$$

$$0 \leq \text{Arc cos } x \leq \pi$$

$$-\frac{\pi}{2} < \text{Arc tan } x < \frac{\pi}{2}$$

## Composite Functions Containing Trigonometric and Inverse Trigonometric Functions

Since Trig. Functions and Inverse Trig. Functions are Inverses

$$\cos(\text{Arc cos } \odot) =$$

Use your knowledge of inverses to solve the equation:

$$\text{Arc sin}(2x) = \frac{\pi}{6}$$

Evaluate  $\cos\left(\text{Arctan}\frac{1}{4}\right)$  without a calculator. Hint: use a right triangle!

Simplify  $\sin(\text{Arccot}(2x))$  Hint: use a right triangle!

Summary: Since the angle is unknown, use the ratio of the value given for the inverse trig function and label two of the sides of a right triangle. Find the missing side then evaluate the trig function asked for.

### Derivatives of Inverse Trigonometric Functions

$$\frac{dy}{dx} \text{Sin}^{-1}u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{dy}{dx} \text{Cos}^{-1}u = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{dy}{dx} \text{Tan}^{-1}u = \frac{1}{1+u^2} \frac{du}{dx}$$

$$\frac{dy}{dx} \text{Cot}^{-1}u = -\frac{1}{1+u^2} \frac{du}{dx}$$

$$\frac{dy}{dx} \text{Sec}^{-1}u = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$$

$$\frac{dy}{dx} \text{Csc}^{-1}u = -\frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$$

EX) Differentiate each function below.

A)  $f(x) = 5 \operatorname{Arcsec}(2x)$

B)  $f(x) = \operatorname{Arcsin} \sqrt{1-4x^2}$

C)  $f(x) = \operatorname{Arccsc} \frac{x}{2}$

D)  $f(x) = \cos(\operatorname{Arc tan}(\sqrt{x}))$

EX) Find the equation of the tangent line of  $f(x) = 3 \operatorname{Arcsin} \sqrt{2x}$  at  $x = \frac{1}{4}$ .