Differentiation of Inverse Trigonometric Functions

W-up: AP Multiple Choice #9 and # 27 (both non-calculator)

"$\text{Arcsin}\,x$" means "the angle whose sine is x

Evaluate

1) $\text{Arcsin}\left(\frac{1}{2}\right)$ 2) $\text{Cos}^{-1}\left(\frac{1}{2}\right)$ 3) $\text{Arctan}\left(\sqrt{3}\right)$

4) $\text{Arcsin}\left(-\frac{1}{2}\right)$ 5) $\text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)$ 6) $\text{Arctan}\left(-\sqrt{3}\right)$

Remember: The answers to inverse trig functions are ANGLES where

$-\frac{\pi}{2} \le \text{Arcsin}\,x \le \frac{\pi}{2}$ $0 \le \text{Arccos}\,x \le \pi$ $-\frac{\pi}{2} < \text{Arctan}\,x < \frac{\pi}{2}$

Composite Functions Containing Trigonometric and Inverse Trigonometric Functions

Since Trig. Functions and Inverse Trig. Functions are Inverses

$\cos(\text{Arccos}\,☺)$=

Use your knowledge of inverses to solve the equation:

$$\text{Arcsin}(2x) = \frac{\pi}{6}$$

Evaluate $\cos\left(\operatorname{Arctan}\frac{1}{4}\right)$ without a calculator. Hint: use a right triangle!

Simplify $\sin\left(\operatorname{Arccot}(2x)\right)$ Hint: use a right triangle!

Summary: Since the angle is unknown, use the ratio of the value given for the inverse trig function and label two of the sides of a right triangle. Find the missing side then evaluate the trig function asked for.

Derivatives of Inverse Trigonometric Functions

$$\frac{dy}{dx}\operatorname{Sin}^{-1}u=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad \frac{dy}{dx}\operatorname{Cos}^{-1}u=-\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{dy}{dx}\operatorname{Tan}^{-1}u=\frac{1}{1+u^2}\frac{du}{dx} \qquad \frac{dy}{dx}\operatorname{Cot}^{-1}u=-\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{dy}{dx}\operatorname{Sec}^{-1}u=\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} \qquad \frac{dy}{dx}\operatorname{Csc}^{-1}u=-\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

EX) Differentiate each function below.

A) $f(x)=5\text{Arcsec}(2x)$

B) $f(x)=\text{Arcsin}\sqrt{1-4x^2}$

C) $f(x)=\text{Arccsc}\frac{x}{2}$

D) $f(x)=\cos\left(\text{Arctan}\left(\sqrt{x}\right)\right)$

EX) Find the equation of the tangent line of $f(x)=3\text{Arcsin}\sqrt{2x}$ at $x=\frac{1}{4}$.