

1) Explain why the function has a zero in the given interval.

1a) $f(x) = \frac{1}{12}x^4 - x^3 + 4$, $[1, 2]$

1b) $f(x) = x^2 - 2 - \cos x$ $[0, \pi]$

$f(x) = \frac{1}{12}x^4 - x^3 + 4$ is continuous on the interval $[1, 2]$. $f(1) = \frac{37}{12}$ and $f(2) = -\frac{8}{3}$. By the Intermediate Value Theorem, there exists a number c in $[1, 2]$ such that $f(c) = 0$.

2) Use the IVT to prove that there must be a zero in the interval $[0, 1]$. Then, use the graphing calculator to find the zero accurate to three decimal places.

2a) $f(x) = x^3 + x - 1$

2b) $g(t) = 2 \cos t - 3t$

$f(x)$ is continuous on $[0, 1]$.

$$f(0) = -1 \text{ and } f(1) = 1$$

By the Intermediate Value Theorem, $f(c) = 0$ for at least one value of c between 0 and 1. Using a graphing utility to zoom in on the graph of $f(x)$, you find that

$x \approx 0.68$. Using the root feature, you find that

$$x \approx 0.6823.$$

2c) The function $f(x) = x^3 - 2x$ is continuous on the closed interval $[-2, 2]$. Prove there are three zeros on the interval using the values in the table below and estimate the location of each.

x	-2	-1	1	2
f(x)	-4	1	-1	4

3a) An airplane flies continuously over a 40 minute time frame with the recorded velocities (in miles/min) given in the table below. Is there any time $0 < t < 40$ at which the airplane is traveling 3 miles per minute? Explain.

t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

YES, we are guaranteed the plane is flying 3 miles per minute at least twice. Since the plane is flying continuously, and flying 4.5 miles per minute at time $t = 20$ and 2.4 miles per minute at $t = 25$ it must be flying 3 miles per minute sometime between 20 and 25 minutes. A second occurrence of this happens over the interval $[30, 35]$ for the same reason.

- 3b) Michelle jogs continuously along a path over a 40 minute time frame with the recorded velocities (in meters/min) given in the table below. Is there any time $0 < t < 40$ at which Michelle is running at 230 meters per minute? Explain.

t (minutes)	0	12	20	40
$v(t)$ (meters per minute)	0	200	240	150

4) AP MULTIPLE CHOICE EXAMPLE

Let f be a continuous function on the closed interval $[-2, 7]$. If $f(-2) = 5$ and $f(7) = -3$, then the Intermediate Value Theorem guarantees that

- (A) $f'(c) = 0$ for at least one c between -2 and 7
- (B) $f'(c) = 0$ for at least one c between -3 and 5
- (C) $f(c) = 0$ for at least one c between -3 and 5
- (D) $f(c) = 0$ for at least one c between -2 and 7

- 5) We want to find $\lim_{x \rightarrow 0} \frac{x}{\sin x}$. Direct substitution and other algebraic methods do not seem to work.

USE YOUR GRAPHING CALCULATOR TO HELP ANSWER THE FOLLOWING QUESTIONS!

A) Estimate $\lim_{x \rightarrow 0} \frac{x}{\sin x}$.

B) Which of the following pairs of functions can be used to verify your answer to part “A” using the squeeze theorem. Explain why or why not for each.

i. $g(x) = \sin x$ and $h(x) = \sec x$

ii. $g(x) = e^x$ and $h(x) = e^{-x}$

iii. $g(x) = |x| + 1$ and $h(x) = -|x| + 1$