AP Calculus Intermediate Value Theorem Critical Homework (Answers)

1b) $f(x) = x^2 - 2 - \cos x$ is continuous on $[0, \pi]$.

$f(0) = -3$ and $f(\pi) = \pi^2 - 1 \approx 8.87 > 0$. By the Intermediate Value Theorem, $f(c) = 0$ for at least one value of c between 0 and π.

2b) $g(t) = 2 \cos t - 3t$

g is continuous on $[0, 1]$.

$g(0) = 2 > 0$ and $g(1) \approx -1.9 < 0$.

By the Intermediate Value Theorem, $g(c) = 0$ for at least one value of c between 0 and 1. Using a graphing utility to zoom in on the graph of $g(t)$, you find that $t \approx 0.56$. Using the root feature, you find that $t \approx 0.5636$.

2c) $f(x)$ is a **continuous** polynomial. All values between -4 and 1 occur on the interval [-2, -1] so a zero must occur in that interval(likely near x = -1). All values between -1 and 1 occur on the interval [-1, 1] so a zero must occur in that interval(likely near x = 0). All values between -1 and 4 occur on the interval [1, 2] so a zero must occur in that interval(likely near x = 1.25).

3b) **YES**, we are guaranteed Michelle jogs 230 meters/minute at least once. Michelle is running 200 meters per minute at time t = 12 minutes & 240 meters per minute at t = 20 minutes. Since all velocities between 200 and 240 must occur on the interval [12, 20] because she jogs **continuously,** we are guaranteed a velocity of 230 meters/minute occurs in this time interval.

4) AP MULTIPLE CHOICE EXAMPLE

Let f be a continuous function on the closed interval $[-2,7]$. If $f(-2)=5$ and $f(7)=-3$, then the Intermediate Value Theorem guarantees that

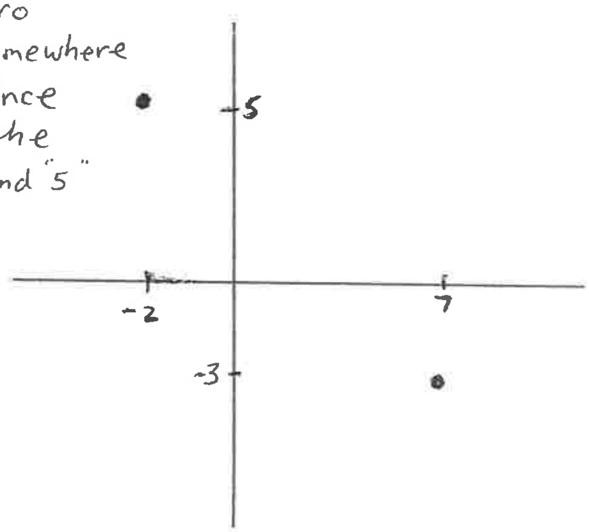
(A) $f'(c)=0$ for at least one c between -2 and 7

(B) $f'(c)=0$ for at least one c between -3 and 5

(C) $f(c)=0$ for at least one c between -3 and 5

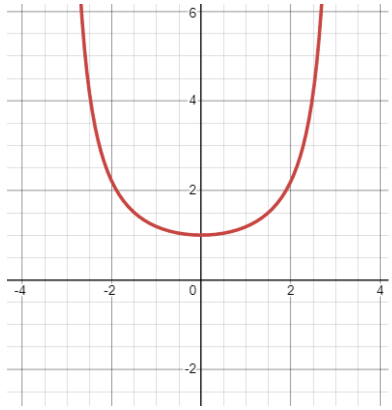
(D) $f(c)=0$ for at least one c between -2 and 7

a y-value of zero is guaranteed somewhere on the interval since zero is between the y-values of "-3" and "5"



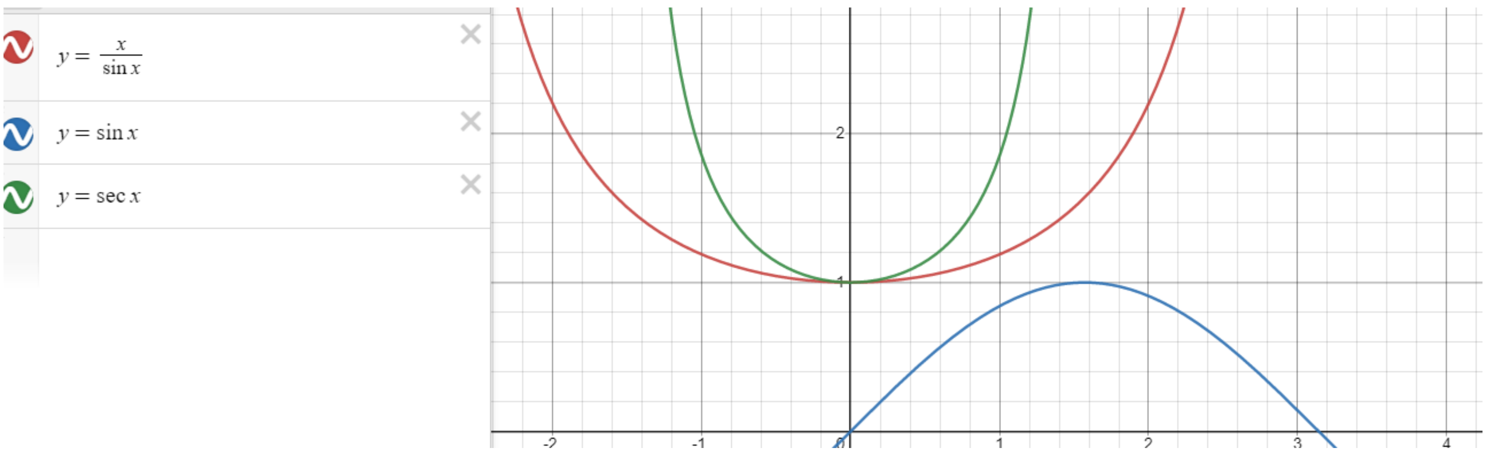
Since continuous over $[-2,7]$, all y-values between -3 and 5 occur on this interval.

A)

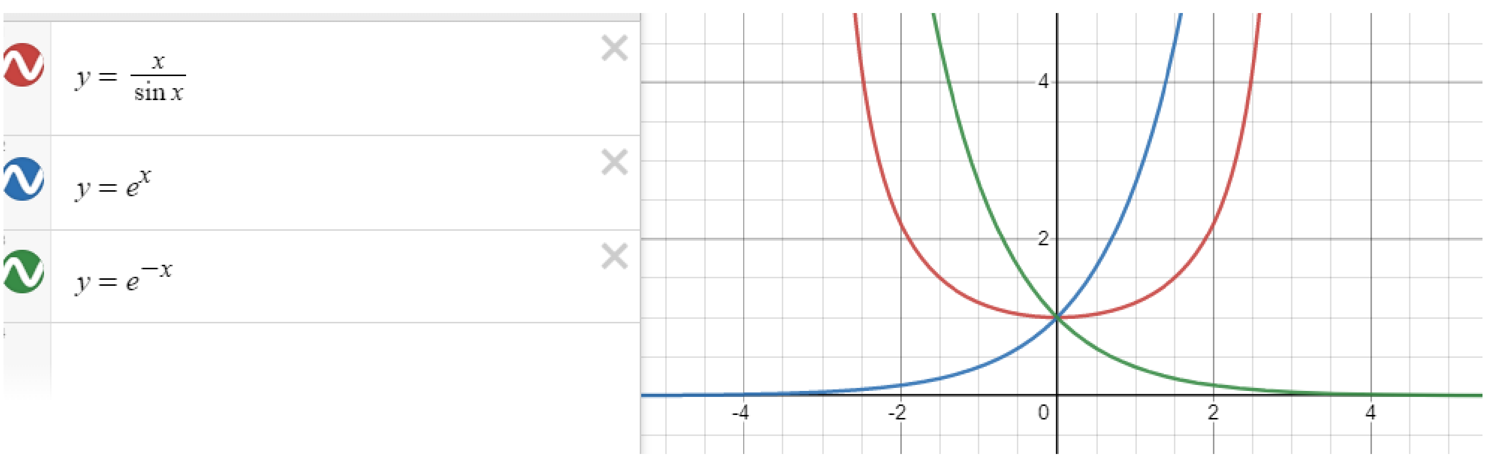


$\lim_{x\to 0}\frac{x}{\sin x}$ Appears to be equal to 1!

B) i. NO, $\sin x \le \frac{x}{\sin x} \le \sec x$ when $x=0$, the $\lim_{x\to 0} \sin x = 0$ while $\lim_{x\to 0} \sec x = 1$



ii. YES, $e^x \le \frac{x}{\sin x} \le e^{-x}$ when $x=0$ $\lim_{x\to 0} e^x = 1$ and $\lim_{x\to 0} e^{-x} = 1$



iii. YES, $-|x|+1 \le \frac{x}{\sin x} \le |x|+1$ when $x=0$ AND $\lim_{x\to 0} -|x|+1=1$ and $\lim_{x\to 0} |x|+1=1$