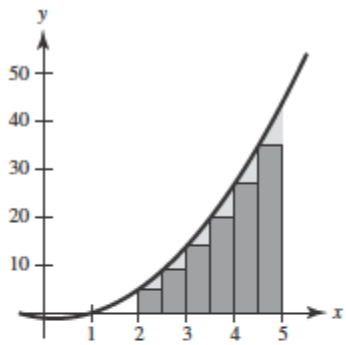
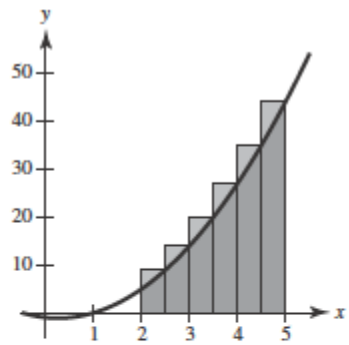
Find the left and right sums used to approximate the area of the region between the graph of the function and the *x*-axis over the given interval, using the given number of rectangles.

1a) $g(x) = 2x^2 - x - 1$, $[2, 5]$, 6 rectangles





$$\Delta x = \frac{5-2}{6} = \frac{1}{2}$$

Left endpoints: Area $\approx \frac{1}{2}[5 + 9 + 14 + 20 + 27 + 35] = 55$

Right endpoints: Area $\approx \frac{1}{2}[9 + 14 + 20 + 27 + 35 + 44] = \frac{149}{2} = 74.5$

1b) $f(x) = 2x + 5$, $[0, 2]$, 4 rectangles

1c) $f(x) = \cos x$, $\left[0, \frac{\pi}{2}\right]$, 4 rectangles

Find the midpoint sum used to approximate the area of the region between the graph of the function and the *x*-axis over the given interval, using 4 rectangles.

2a) $f(x) = \tan x, \quad \left[0, \frac{\pi}{4}\right]$

2b) $f(x) = x^2 + 3, \quad [0, 2]$

Let $c_i = \frac{x_i + x_{i-1}}{2}$.

$\Delta x = \frac{\pi}{16}, \; c_1 = \frac{\pi}{32}, \; c_2 = \frac{3\pi}{32}, \; c_3 = \frac{5\pi}{32}, \; c_4 = \frac{7\pi}{32}$

Area $= \frac{\pi}{16}\left(\tan\frac{\pi}{32} + \tan\frac{3\pi}{32} + \tan\frac{5\pi}{32} + \tan\frac{7\pi}{32}\right) \approx 0.345$

Find the trapezoid sum used to approximate the area of the region between the graph of the function and the *x*-axis over the given interval, using the given number of trapezoids.

3a) $f(x) = x^3, \; [0,2], \;$ 4 trapezoids

3b) $f(x) = \sqrt{x}, \; [4,9], \;$ 8 trapezoids

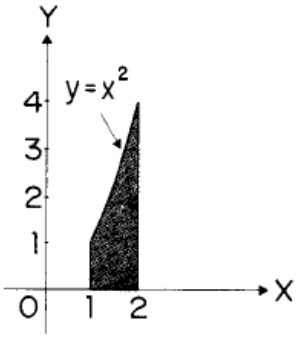
Area $\approx \frac{1}{4}\left[0 + 2\left(\frac{1}{2}\right)^3 + 2(1)^3 + 2\left(\frac{3}{2}\right)^3 + (2)^3\right] = \frac{17}{4}$

3c) $f(x) = \sqrt{1+x^3}, \; [0,2], \;$ 4 trapezoids

3d) $f(x) = x\tan x, \; \left[0, \frac{\pi}{4}\right], \;$ 4 trapezoids

4) AP MULTIPLE CHOICE EXAMPLES

1)



Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x = \frac{4}{3}$ and $x = \frac{5}{3}$.

(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

2)

x	1	3	5	8	10
$f(x)$	7	12	16	23	17

The function f is continuous on the closed interval $[1,10]$ and has values as shown in the table above. Using a right Riemann sum with four subintervals $[1,3]$, $[3,5]$, $[5,8]$, $[8,10]$, what is the area under $f(x)$ on the interval $[1, 10]$.

(A) 96 (B) 116 (C) 132 (D) 159