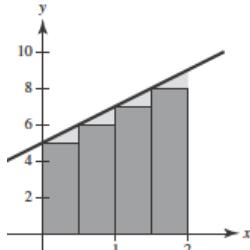


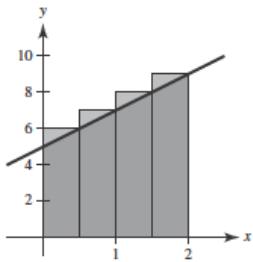
1b)



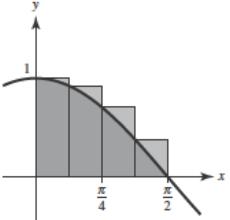
$$\Delta x = \frac{2 - 0}{4} = \frac{1}{2}$$

Left endpoints: Area $\approx \frac{1}{2}[5 + 6 + 7 + 8] = \frac{26}{2} = 13$

Right endpoints: Area $\approx \frac{1}{2}[6 + 7 + 8 + 9] = \frac{30}{2} = 15$



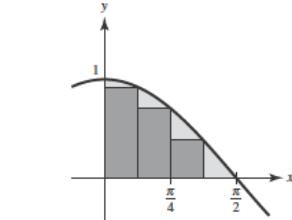
1c)



$$\Delta x = \frac{\pi/2 - 0}{4} = \frac{\pi}{8}$$

Left endpoints: Area $\approx \frac{\pi}{8} \left[\cos(0) + \cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{3\pi}{8}\right) \right] \approx 1.1835$

Right endpoints: Area $\approx \frac{\pi}{8} \left[\cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{3\pi}{8}\right) + \cos\left(\frac{\pi}{2}\right) \right] \approx 0.7908$

2b) Let $c_i = \frac{x_i + x_{i-1}}{2}$.

$$\Delta x = \frac{1}{2}, c_1 = \frac{1}{4}, c_2 = \frac{3}{4}, c_3 = \frac{5}{4}, c_4 = \frac{7}{4}$$

$$\text{Area} \approx \frac{1}{2} \left[\left(\frac{1}{16} + 3 \right) + \left(\frac{9}{16} + 3 \right) + \left(\frac{25}{16} + 3 \right) + \left(\frac{49}{16} + 3 \right) \right] = \frac{69}{8}$$

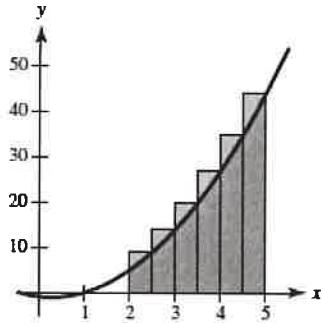
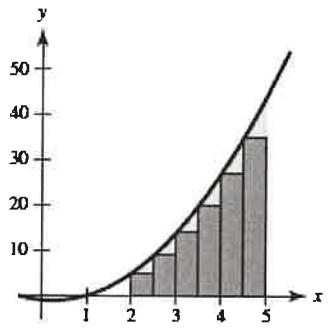
$$3b) \text{Area} \approx \frac{5}{16} \left[2 + 2\sqrt{\frac{37}{8}} + 2\sqrt{\frac{21}{4}} + 2\sqrt{\frac{47}{8}} + 2\sqrt{\frac{26}{4}} + 2\sqrt{\frac{57}{8}} + 2\sqrt{\frac{31}{4}} + 2\sqrt{\frac{67}{8}} + 3 \right] \approx 12.6640$$

$$3c) \text{Area} \approx \frac{1}{4} \left[1 + 2\sqrt{1 + \left(\frac{1}{8}\right)} + 2\sqrt{2} + 2\sqrt{1 + \left(\frac{27}{8}\right)} + 3 \right] \approx 3.283$$

$$3d) \text{Area} \approx \frac{\pi}{32} \left[0 + 2\left(\frac{\pi}{16}\right) \tan\left(\frac{\pi}{16}\right) + 2\left(\frac{2\pi}{16}\right) \tan\left(\frac{2\pi}{16}\right) + 2\left(\frac{3\pi}{16}\right) \tan\left(\frac{3\pi}{16}\right) + \frac{\pi}{4} \right] \approx 0.194$$

Find the left and right sums used to approximate the area of the region between the graph of the function and the x -axis over the given interval, using the given number of rectangles.

1a) $g(x) = 2x^2 - x - 1$, $[2, 5]$, 6 rectangles



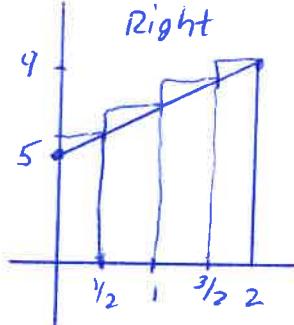
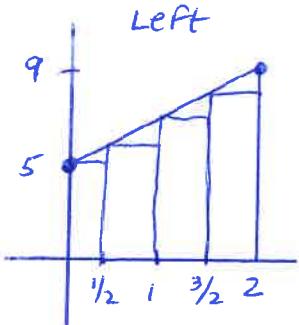
$$\Delta x = \frac{5-2}{6} = \frac{1}{2}$$

Left endpoints: Area $\approx \frac{1}{2}[5 + 9 + 14 + 20 + 27 + 35] = 55$

Right endpoints: Area $\approx \frac{1}{2}[9 + 14 + 20 + 27 + 35 + 44] = \frac{149}{2} = 74.5$

1b) $f(x) = 2x + 5$, $[0, 2]$, 4 rectangles

1c) $f(x) = \cos x$, $\left[0, \frac{\pi}{2}\right]$, 4 rectangles



$$\Delta x = \frac{2-0}{4} = \frac{1}{2}$$

$$\frac{1}{2} (f(0) + f(\frac{1}{2}) + f(1) + f(\frac{3}{2}))$$

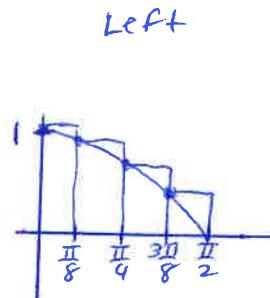
$$\frac{1}{2} (5 + 6 + 7 + 8)$$

(13)

$$\frac{1}{2} (f(\frac{1}{2}) + f(1) + f(\frac{3}{2}) + f(2))$$

$$\frac{1}{2} (6 + 7 + 8 + 9)$$

(15)

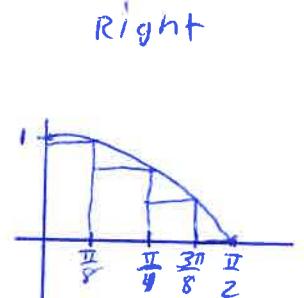


$$\Delta x = \frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$$

$$\frac{\pi}{8} (f(0) + f(\frac{\pi}{8}) + f(\frac{\pi}{4}) + f(\frac{3\pi}{8}))$$

$$\frac{\pi}{8} (\overbrace{1 + .924 + .707 + .382}^{\approx 1.1835})$$

≈ 1.1835



$$\frac{\pi}{8} (f(\frac{\pi}{8}) + f(\frac{\pi}{4}) + f(\frac{3\pi}{8}) + f(\frac{\pi}{2}))$$

$$\frac{\pi}{8} (.924 + .707 + .382 + 0)$$

$\approx .7908$

Find the midpoint sum used to approximate the area of the region between the graph of the function and the x -axis over the given interval, using 4 rectangles.

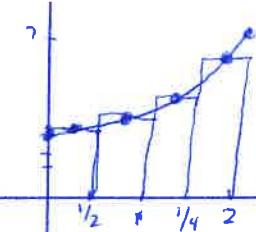
2a) $f(x) = \tan x, \left[0, \frac{\pi}{4}\right]$

Let $c_i = \frac{x_i + x_{i-1}}{2}$.

$$\Delta x = \frac{\pi}{16}, c_1 = \frac{\pi}{32}, c_2 = \frac{3\pi}{32}, c_3 = \frac{5\pi}{32}, c_4 = \frac{7\pi}{32}$$

$$\text{Area} = \frac{\pi}{16} \left(\tan \frac{\pi}{32} + \tan \frac{3\pi}{32} + \tan \frac{5\pi}{32} + \tan \frac{7\pi}{32} \right) \approx 0.345$$

2b) $f(x) = x^2 + 3, [0, 2], \Delta x = \frac{2-0}{4} = \frac{1}{2}$



$$A = \frac{1}{2} \left(f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) + f\left(\frac{5}{4}\right) + f\left(\frac{7}{4}\right) \right)$$

$$= \frac{1}{2} \left(\frac{1}{16} + 3 + \frac{9}{16} + 3 + \frac{25}{16} + 3 + \frac{49}{16} + 3 \right)$$

$$= \frac{1}{2} (12 + \frac{84}{16})$$

$$= 6 + \frac{42}{16}$$

$$= \boxed{\frac{69}{8}}$$

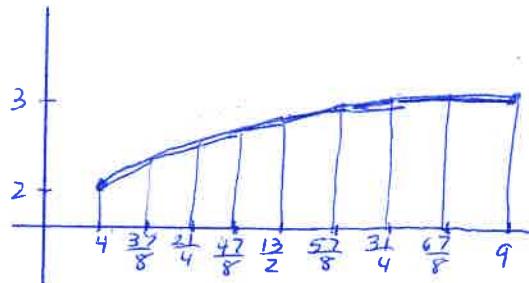
Find the trapezoid sum used to approximate the area of the region between the graph of the function and the x -axis over the given interval, using the given number of trapezoids.

3a) $f(x) = x^3, [0, 2], 4$ trapezoids

$$\text{Area} \approx \frac{1}{4} \left[0 + 2\left(\frac{1}{2}\right)^3 + 2(1)^3 + 2\left(\frac{3}{2}\right)^3 + (2)^3 \right] = \frac{17}{4}$$

3b) $f(x) = \sqrt{x}, [4, 9], 8$ trapezoids

$$\Delta x = \frac{9-4}{8} = \frac{5}{8}$$

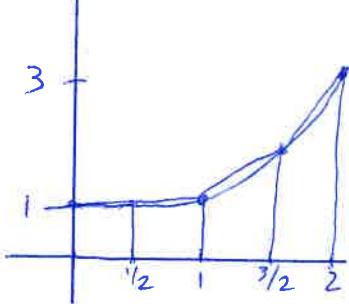


$$A \approx 12.664$$

$$A \approx \frac{5}{16} \left(f(4) + 2f\left(\frac{37}{8}\right) + 2f\left(\frac{21}{4}\right) + 2f\left(\frac{47}{8}\right) + 2f\left(\frac{13}{2}\right) + 2f\left(\frac{57}{8}\right) + 2f\left(\frac{31}{4}\right) + 2f\left(\frac{67}{8}\right) + 2f\left(\frac{9}{2}\right) + f(9) \right)$$

3c) $f(x) = \sqrt{1+x^3}, [0, 2], 4$ trapezoids

$$\Delta x = \frac{2-0}{4} = \frac{1}{2}$$

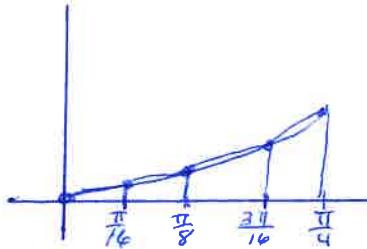


$$A = \frac{1}{4} \left(f(0) + 2f\left(\frac{1}{2}\right) + 2f(1) + 2f\left(\frac{3}{2}\right) + f(2) \right)$$

$$\approx \boxed{3.283}$$

3d) $f(x) = x \tan x, \left[0, \frac{\pi}{4}\right], 4$ trapezoids

$$\Delta x = \frac{\pi/4 - 0}{4} = \frac{\pi}{16}$$

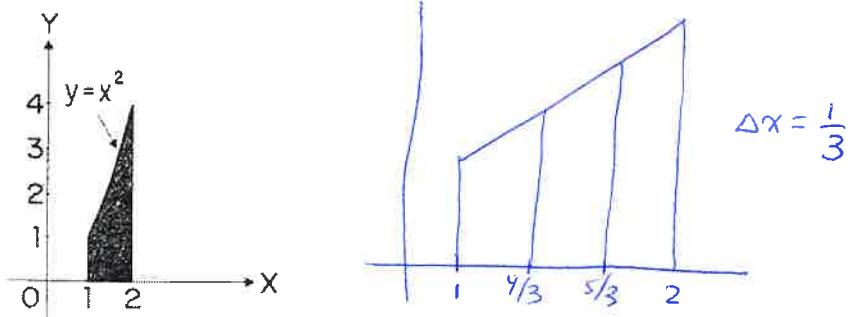


$$A = \frac{\pi}{32} \left(f(0) + 2f\left(\frac{\pi}{16}\right) + 2f\left(\frac{3\pi}{16}\right) + 2f\left(\frac{7\pi}{16}\right) + f\left(\frac{\pi}{4}\right) \right)$$

$$\approx \boxed{1.94}$$

4) AP MULTIPLE CHOICE EXAMPLES

1)



Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x = \frac{4}{3}$ and $x = \frac{5}{3}$.

- (A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

$$\begin{aligned} A &= \frac{[f(1) + f(\frac{4}{3})] \cdot \frac{1}{3}}{2} + \frac{[f(\frac{4}{3}) + f(\frac{5}{3})] \cdot \frac{1}{3}}{2} + \frac{[f(\frac{5}{3}) + f(2)] \cdot \frac{1}{3}}{2} \\ &= \frac{1}{6} [f(1) + 2f(\frac{4}{3}) + 2f(\frac{5}{3}) + f(2)] \\ &= \frac{1}{6} [1 + \frac{32}{9} + \frac{50}{9} + 4] \\ &= \frac{1}{6} [\frac{127}{9}] = \frac{127}{54} \end{aligned}$$

2)

x	1	3	5	8	10
$f(x)$	7	12	16	23	17

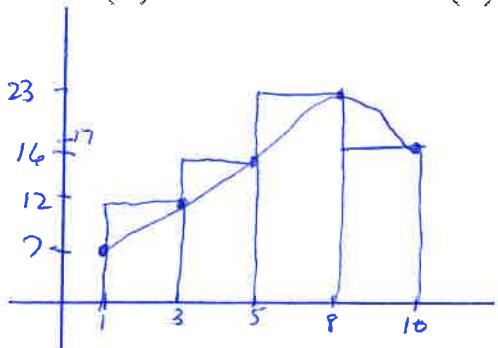
The function f is continuous on the closed interval $[1, 10]$ and has values as shown in the table above. Using a right Riemann sum with four subintervals $[1, 3]$, $[3, 5]$, $[5, 8]$, $[8, 10]$, what is the area under $f(x)$ on the interval $[1, 10]$.

- (A) 96

- (B) 116

- (C) 132

- (D) 159



$$2 \cdot f(3) + 2 \cdot f(5) + 3 \cdot f(8) + 2 \cdot f(10)$$

$$2 \cdot (12) + 2 \cdot (16) + 3 \cdot (23) + 2 \cdot (17)$$

$$24 + 32 + 69 + 34$$

159