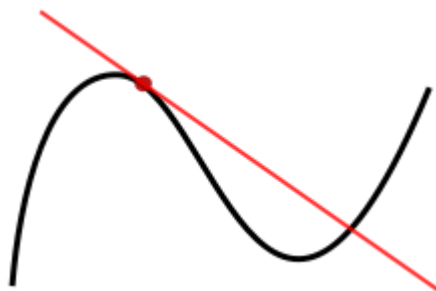


The Limit Definition of Derivative

w-up: Write an equation of a line traveling through $(-3, 4)$ with slope of $\frac{1}{2}$.

Tangent line: a straight line that "just touches" the curve at that point. (looks like a see-saw)

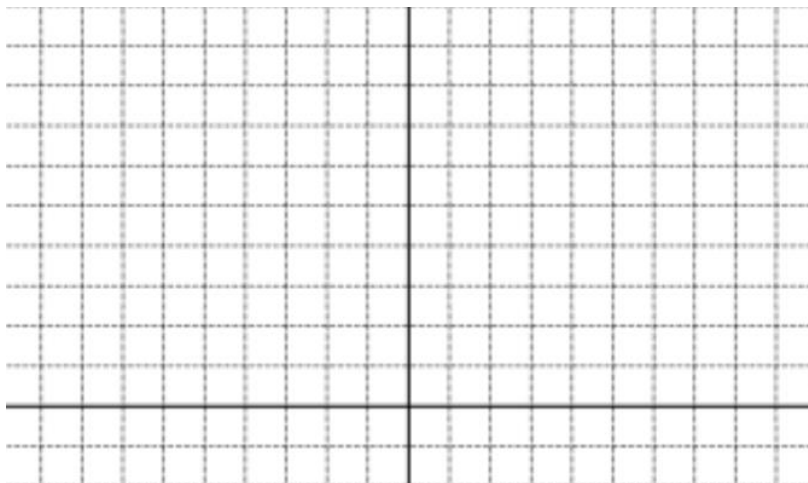


Draw five more tangent lines on the curve above.

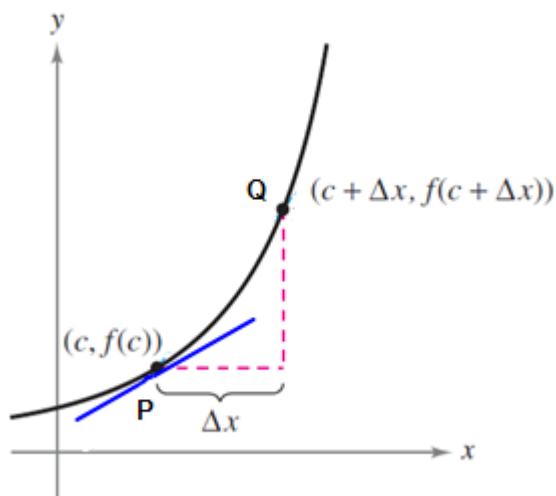
The **slope of the tangent line** represents the "steepness" of any curve at any point and we call this steepness the **Instantaneous Rate of Change** of a function at any point.

Question: What kind of functions is the instantaneous rate of change constant(always the same)?

EX) Sketch the graph of $y = \frac{1}{4}x^2$ on graph paper and estimate the slope of the tangent line at $x = 2$, $x = 4$ and $x = -1$



Finding the exact slope of a tangent line using limits



Let point P be any point where the Rate of Change (slope) is to be found.

Let point Q be any point on the function Δx away from the x-coordinate of point P.

Point P $(c, f(c))$ **Point Q** $(c + \Delta x, f(c + \Delta x))$

Write an expression using these coordinates to find the slope of \overline{PQ} .

Note: the closer point Q is to point P (so as Δx gets closer to zero) the closer the slope of the secant is to the actual slope of the tangent line at point P.

$$\text{Slope of the Tangent line} = \lim_{\Delta x \rightarrow 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

where c is the x -value constant we want to find slope at!

So, $\lim_{\Delta x \rightarrow 0} \frac{f(2 + \Delta x) - f(2)}{\Delta x}$ using the function $f(x) = \frac{1}{4}x^2$ will find the slope of the tangent line to $f(x)$ at the point $(2,1)$. Find the exact value of the slope at this point.

Evaluate $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$ *without* substitution for the x -value. This will result in an algebraic expression instead of a value. This will serve as a “slope finding formula” for finding the slope of a tangent line at **any** x -value!

Use this “slope finding formula” to find the slope of the tangent line to

$$f(x) = \frac{1}{4}x^2 \text{ at } x = 4 \text{ and } x = -1$$

This “slope finding formula” is known as the **DERIVATIVE**

Derivative: Functional expression which will find the Rate of Change (slope of the tangent line) for any function at any point

Note: The process of finding a derivative is called “differentiation”

Notation

$f'(x)$ (read f prime of x)

$\frac{dy}{dx}$ (read dy, dx)

y'

$\frac{d}{dx} f(x)$

EX) Find the equation of the tangent line to $f(x) = \frac{1}{4}x^2$ at $x = -3$

Alternate Forms for the Limit Definition of Derivative

Using "h" instead of Δx

$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ finds derivative at any x -value

AP EXAMPLES

Explain the meaning of each limit.

A) $\lim_{\Delta x \rightarrow 0} \frac{\cos(x+\Delta x) - \cos x}{\Delta x}$

B) $\lim_{\Delta x \rightarrow 0} \frac{\tan\left(\frac{\pi}{4} + \Delta x\right) - \tan\frac{\pi}{4}}{\Delta x}$

C) $\lim_{\Delta x \rightarrow 0} \frac{2(3+\Delta x)^3 - 2(3)^3}{\Delta x}$