

Write *but do not evaluate* a limit that would find the derivative for each function.

1a) $f(x) = x^2 + x - 3$

1b) $f(x) = x^3 - 12x$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 + (x + \Delta x) - 3 - (x^2 + x - 3)}{\Delta x} \end{aligned}$$

1c) $f(x) = \sqrt{x + 4}$

1d) $f(x) = \frac{4}{\sqrt{x}}$

1e) $f(x) = \tan x$

Find the derivative by the limit process. Then, use the derivative to find the slope of the tangent line at the given point. Lastly, write the equation of the tangent line to the function at that given point.

2a) $f(x) = x^2 + x - 3$, point (3, 9)

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 + (x + \Delta x) - 3 - (x^2 + x - 3)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 + x + \Delta x - 3 - x^2 - x + 3}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2x(\Delta x) + (\Delta x)^2 + \Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} (2x + \Delta x + 1) = 2x + 1 \end{aligned}$$

$$f'(x) = 2x + 1$$

$$f'(3) = 7$$

Equation of tangent line with slope 7
travelling through (3, 9)

is $y - 9 = 7(x - 3)$ or $y = 7x - 12$

2b) $f(x) = x^2 + 3, (1, 4)$

2c) $f(x) = \sqrt{x}, (1, 1)$